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RESEARCH

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Optical soliton perturbation of Fokas-Lenells equation by the Laplace-Adomian decomposition algorithm

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Abstract

This paper displays numerical simulation for bright and dark optical solitons that emerge from Fokas-Lenells equation which is studied in the context of dispersive solitons in polarization-preserving fibers. The Laplace-Adomian decomposition scheme is the numerical tool adopted in the paper. The numerical results, for bright and dark solitons, are expository and therefore supplement the analytical developments, thus far.

Keywords: Fokas-lenells equation, Polarization-preserving fibers, Adomian decomposition method, Optical solitons solutions, Perturbation

Introduction

One of the governing models to study dispersive solitons is Fokas-Lennels equation (FLE) [1–13]. In such a model, in addition to group velocity dispersion (GVD), one considers, inter-modal dispersion as well as nonlinear dispersion thus treating it with a flavor of additional dispersive effects. There has been a plethora of analytical tools that have been implemented to study FLE. They range from semi-inverse variational principle, Lie symmetry analysis, Riccati equation approach, exp-function method, traveling wave hypothesis, trial function method and further wide varieties. This paper will be changing gears to study the model from a different perspective. One of the very many and modern numerical algorithms that will be implemented is the Laplace-Adomian decomposition integration scheme. This method has been successfully applied to variety of other models from optics [14–16]. This paper now studies FLE, for the first time, by the aid of Laplace-Adomian decomposition scheme. The details are sketched in the remainder of the paper, after introducing the model.

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The Fokas-Lenells equation (FLE) in presence of perturbation terms

The dimensionless form of the perturbed Fokas-Lenells equation (FLE) is given by

$$\begin{aligned} &iu_t + a_1 u_{xx} + a_2 u_{xt} + |u|^2 \left(bu + i\sigma u_x\right) \\ &\quad = i\left[\alpha u_x + \lambda \left(|u|^2 u\right)_x + \mu \left(|u|^2\right)_x u\right]. \end{aligned} \tag{1}$$

This equation was first studied in [17–24] and arises in various systems such as water waves, plasma physics, solid state physics and nonlinear optics. In Eq. (1), $u(x,t)$ represents a complex field envelope, and x and t are spatial and temporal variables, respectively. Here, the coefficient a_1 is the group velocity dispersion (GVD) and a_2 is the spatio-temporal dispersion (STD) the coefficient b is self-phase modulation moreover σ accounts for nonlinear dispersion. In the perturbative term of Eq. (1), the first term represents the inter-modal dispersion (IMD), the second term is the self-steepening effect and finally the last term accounts for another version of nonlinear dispersion (ND).

Bright optical solitons

The bright optical soliton solution to (1) is given by [5, 11]:

$$u(x,t) = A\,\text{sech}\left[(x - \nu t)\right] e^{i[-\kappa x + \omega t + \theta]}. \tag{2}$$

Here, ν is the soliton velocity, κ is the soliton frequency, ω is the angular velocity and θ is the phase center.

The amplitude A of the soliton in this case is given by

$$A = \pm\sqrt{\frac{2(a_1 - a_2\nu)}{b - \kappa\lambda + \kappa\sigma}}, \tag{3}$$

where, the velocity of the soliton in relation to the coefficients that appear in the Eq. (1) is

$$\nu = \frac{\alpha + 2a_1\kappa - a_2\omega}{a_2\kappa - 1}, \tag{4}$$

and the constraints conditions on the parameters are

$$a_2\kappa \neq 1, \quad 3\lambda + 2\mu - \sigma = 0. \tag{5}$$

In the previous context κ is any parameter that satisfies the Eq. (5).

Dark optical solitons

noindent The dark optical soliton solution to (1) is given by [5, 11]:

$$u(x,t) = B\tanh\left[(x - \nu t)\right] e^{i[-\kappa x + \omega t + \theta]}. \tag{6}$$

Here, ν is the soliton velocity, κ is the soliton frequency, ω is the angular velocity and θ is the phase center.
The amplitude B of the soliton in this case is given by

$$B = \pm\sqrt{\frac{-2(a_1 - a_2\nu)}{b - \kappa\lambda + \kappa\sigma}}, \tag{7}$$

where, the velocity of the soliton in relation to the coefficients that appear in the Eq. (1) is

$$\nu = \frac{\alpha + 2a_1\kappa - a_2\omega}{a_2\kappa - 1}, \tag{8}$$

and the constraints conditions on the parameters are

$$a_2\kappa \neq 1, \quad 3\lambda + 2\mu - \sigma = 0. \tag{9}$$

In the previous context κ is any parameter that satisfies the Eq. (9).

The Laplace Adomian Decomposition Method (LADM)

To illustrate the basic concept of Laplace-Adomian decomposition algorithm, we consider the general form of second order nonlinear partial differential equations in the form

$$F\left(u(x,t)\right) = 0, \tag{10}$$

with initial conditions

$$u(x,0) = f(x), \quad u_x(x,0) = g(x). \tag{11}$$

where F is a differential operator. Now, let us decompose this operator as $F = L+R+N$ where $L(u) = \frac{\partial u}{\partial t}$ stands for a linear differential operator. The operators R and N are the remaining linear and nonlinear parts, respectively.
With these considerations, Eq. (10) can now be rewritten as

$$Lu(x,t) = Ru(x,t) + Nu(x,t). \tag{12}$$

Solving for $Lu(x,t)$ and applying the Laplace transform respect to t to Eq. (12), gives

$$\mathcal{L}\left\{Lu(x,t)\right\} = \mathcal{L}\left\{Ru(x,t) + Nu(x,t)\right\}. \tag{13}$$

Thus, Eq. (13) turns out to be equivalent to

$$su(x,s) - u(x,0) = \mathcal{L}\left\{Ru(x,t) + Nu(x,t)\right\}. \tag{14}$$

Using Eq. (11), one get

$$u(x,s) = \frac{f(x)}{s} + \frac{1}{s}\mathcal{L}\left\{Ru(x,t) + Nu(x,t)\right\}. \tag{15}$$

Finally, by applying inverse Laplace transformation $\mathcal{L}^{-1}$ on both sides of the Eq. (15), we obtain

$$u(x,t) = f(x) + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{Ru(x,t) + Nu(x,t)\}\right]. \tag{16}$$

The Laplace-Adomian decomposition algorithm assumes the solution $u(x,t)$ can be expanded into infinite series given by

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{17}$$

Moreover, Also the nonlinear operator N is decomposed as

$$Nu(x,t) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \ldots, u_n),. \tag{18}$$

Each A_n is an Adomian polynomial of $u_0, u_1, \ldots, u_n$ that can be calculated for all forms of nonlinearity according to the following formula [25–27]:

$$A_0 = N(u_0),$$

$$A_n = \frac{1}{n}\sum_{i=1}^{m}\sum_{k=0}^{n-1}(k+1)u_{i,k+1}\frac{\partial}{\partial u_{i,0}}A_{n-1-k}, \quad n \geq 1. \tag{19}$$

Therefore Adomian's polynomials are given by
$A_0 = N(u_0)$
$A_1 = u_1N'(u_0)$
$A_2 = u_2N'(u_0) + \frac{1}{2}u_1^2N''(u_0)$
$A_3 = u_3N'(u_0) + u_1u_2N''(u_0) + \frac{1}{3!}u_1^3N^{(3)}(u_0)$
$A_4 = u_4N'(u_0) + \left(\frac{1}{2}u_2^2 + u_1u_3\right)N''(u_0) + \frac{1}{2!}u_1^2u_2N^{(3)}(u_0) + \frac{1}{4!}u_1^4N^{(4)}(u_0)$
$\vdots$
All other polynomials are calculated in a similar way.
Substituting (17) and (18) into Eq. (16) gives rise to

$$\sum_{n=0}^{\infty} u_n(x,t) = f(x) + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left\{R\sum_{n=0}^{\infty}u_n(x,t) + \sum_{n=0}^{\infty}A_n(u_0, u_1, \ldots, u_n)\right\}\right]. \tag{20}$$

Table 1 Bright optical solitons

Cases	a_1	a_2	b	σ	α	λ	μ	ν	κ	A	N	|Max Error|
1	1.00	0.50	2.00	1.00	2.00	1.00	−1.00	−1.66	0.50	0.27	12	2.10×10^{-10}
2	3.00	0.16	1.00	1.00	2.00	1.00	−1.00	−3.47	0.25	2.67	12	3.00×10^{-10}
3	1.50	0.25	3.00	1.00	1.00	−1.00	2.00	−5.00	1.00	0.31	12	1.50×10^{-10}
4	0.20	0.33	4.00	2.00	2.00	2.00	−2.00	−7.40	2.00	1.45	12	1.00×10^{-10}

Hence, Eq. (20) suggests the following iterative algorithm

$$\begin{cases} u_0(x,t) = f(x), \\ u_{n+1}(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left\{Ru_n(x,t) + A_n(u_0,u_1,\ldots,u_n)\right\}\right], \quad n = 0,1,2,\ldots \end{cases} \tag{21}$$

Finally, after determining u_n's, the N-term truncated approximation of the solution is obtained as

$$u_N(x,t) = \sum_{n=0}^{N-1} u_n(x,t), \quad N \geq 1. \tag{22}$$

From this analysis it is evident that, the Adomian decomposition method, combined with the Laplace transform requires less effort in comparison with the traditional Adomian decomposition method. This method considerably decreases the number of calculations. In addition, Adomian decomposition procedure is easily established without requiring to linearize the problem.

Solution of the perturbed Fokas-Lenells equation by LADM

In this section, we outline the application of LADM to obtain explicit solution of Eq. (1) with the initial conditions $u(x,0) = f(x),\ u_x(x,0) = g(x)$.

Let us consider the dimensionless form of the perturbed Fokas-Lenells equation Eq. (1) in an operator form

$$Lu(x,t)+Ru(x,t)+N_1u(x,t)+N_2u(x,t)+N_3u(x,t) = 0 \tag{23}$$

where the notation $N_1u = -i|u|^2(bu + i\sigma u_x)$, $N_2u = -\lambda(|u|^2u)_x$ and $N_3u = -\mu u(|u|^2)_x$ symbolize the nonlinear term, respectively. The notation $Ru = -(\alpha u_x + ia_1u_{xx} + ia_2u_{xt})$ symbolize the linear differential operator and $Lu = u_t$ simply means derivative with respect to time.
The LADM represents solution as an infinite series of components given below,

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{24}$$

The nonlinear terms N_1u, N_2u and N_3u can be decomposed into infinite series of Adomian polynomials given by:

$$N_1u = -i|u|^2(bu + i\sigma u_x) = \sum_{n=0}^{\infty} P_n(u_0,u_1,\ldots,u_n), \tag{25}$$

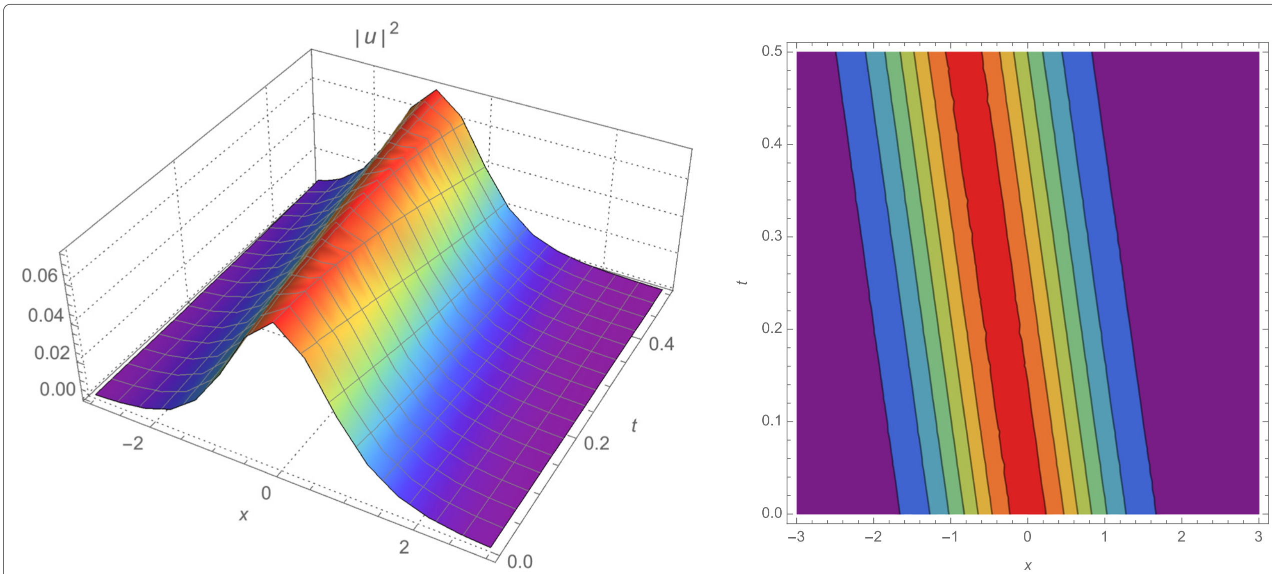

Fig. 1 Dynamic evolution profile of $|u|^2$ via LADM (left) and contour plot of the wave amplitude of $|u|^2$ (right) for the values of the parameters used in case 1 with |Max Error| = 2.1×10^{-10}

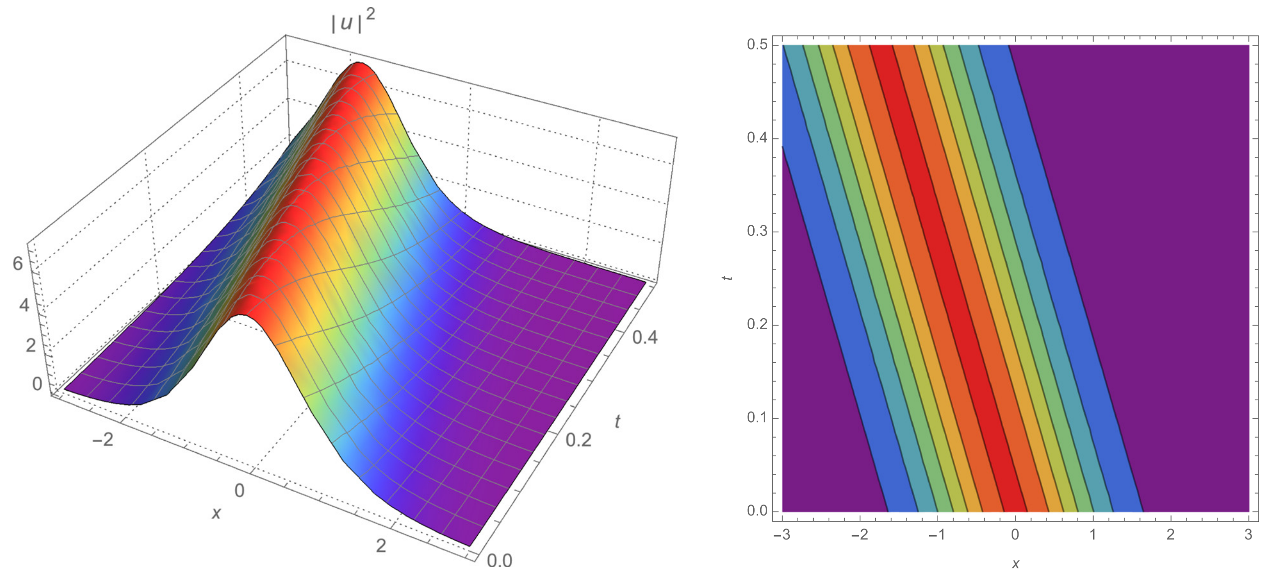

Fig. 2 Dynamic evolution profile of $|u|^2$ via LADM (left) and contour plot of the wave amplitude of $|u|^2$ (right) for the values of the parameters used in case 2 with $|\text{Max Error}| = 3.0 \times 10^{-10}$

$$N_2u = -\lambda \left(|u|^2u\right)_x = \sum_{n=0}^{\infty} Q_n(u_0, u_1, \ldots, u_n), \qquad (26)$$

and

$$N_3u = -\mu u \left(|u|^2\right)_x = \sum_{n=0}^{\infty} R_n(u_0, u_1, \ldots, u_n). \qquad (27)$$

Here P_n, Q_n and R_n are the Adomian polynomials and can be calculated by the formula given by the Eq. (19), that is,

$$P_0 = N_1(u_0), \quad Q_0 = N_2(u_0), \quad R_0 = N_3(u_0),$$

and for every $n \geq 1$ we have

$$P_n = \frac{1}{n} \sum_{i=1}^{m} \sum_{k=0}^{n-1} (k+1) u_{i,k+1} \frac{\partial}{\partial u_{i,0}} P_{n-1-k}, \qquad (28)$$

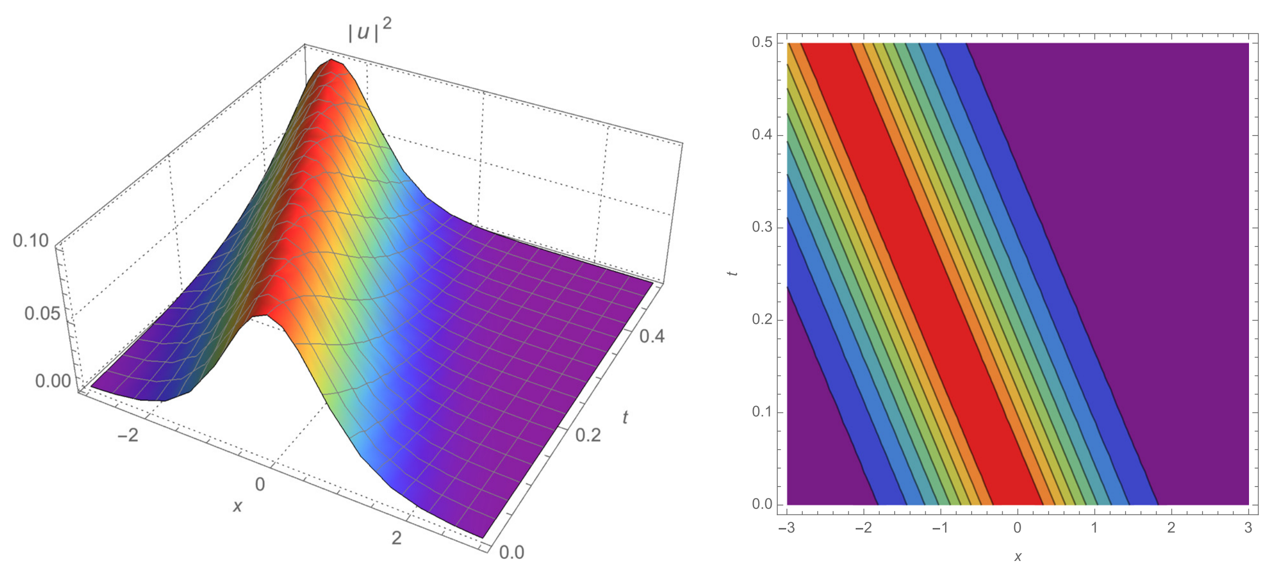

Fig. 3 Dynamic evolution profile of $|u|^2$ via LADM (left) and contour plot of the wave amplitude of $|u|^2$ (right) for the values of the parameters used in case 3 with $|\text{Max Error}| = 1.5 \times 10^{-10}$

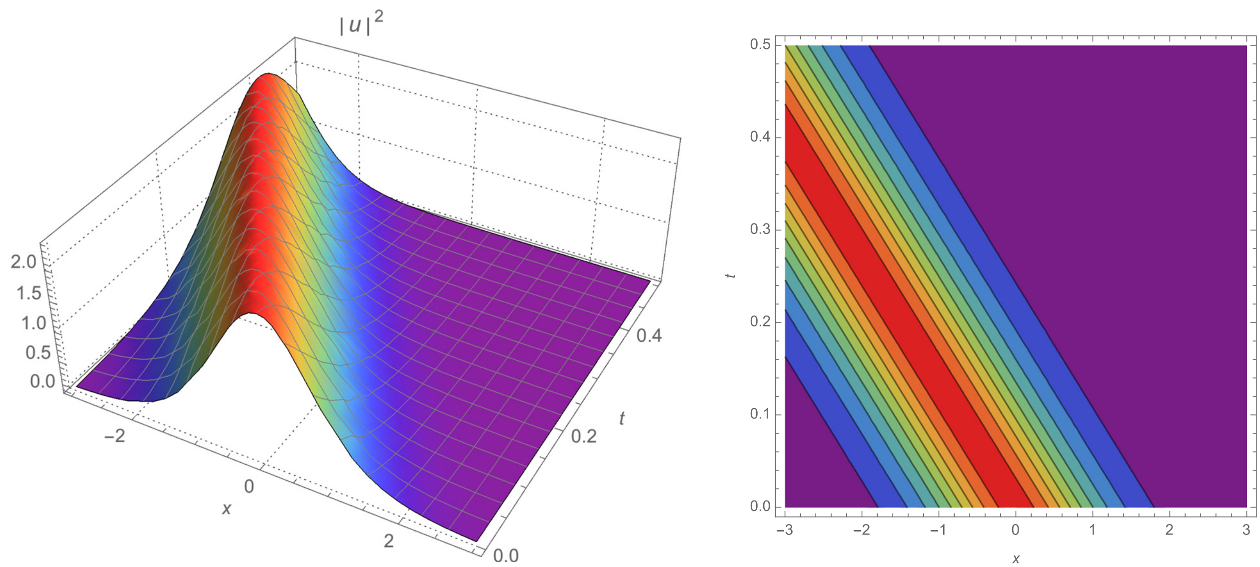

Fig. 4 Dynamic evolution profile of $|u|^2$ via LADM (left) and contour plot of the wave amplitude of $|u|^2$ (right) for the values of the parameters used in case 4 with $|\text{Max Error}| = 1.0 \times 10^{-10}$

$$Q_n = \frac{1}{n}\sum_{i=1}^{m}\sum_{k=0}^{n-1}(k+1)u_{i,k+1}\frac{\partial}{\partial u_{i,0}}Q_{n-1-k}, \tag{29}$$

$$R_n = \frac{1}{n}\sum_{i=1}^{m}\sum_{k=0}^{n-1}(k+1)u_{i,k+1}\frac{\partial}{\partial u_{i,0}}R_{n-1-k}. \tag{30}$$

The first few Adomian polynomials are given by

$$\begin{aligned}
P_0 &= -ibu_0^2\bar{u}_0,\\
P_1 &= -2ibu_0u_1\bar{u}_0 - ibu_0^2\bar{u}_1,\\
P_2 &= -2ibu_0u_2\bar{u}_0 - ibu_1^2\bar{u}_0 - 2ibu_0u_1\bar{u}_1 - ibu_0^2\bar{u}_2,\\
P_3 &= -2ibu_0u_3\bar{u}_0 - 2ibu_1u_2\bar{u}_0 - 2ibu_0u_2\bar{u}_1 - ibu_1^2\bar{u}_1\\
&\quad - 2ibu_0u_1\bar{u}_2 - ibu_0^2\bar{u}_3,\\
P_4 &= -ib\bar{u}_0u_2^2 - 2ibu_0\bar{u}_0u_4 - 2ib\bar{u}_0u_1u_3 - 2ibu_0\bar{u}_1u_3\\
&\quad - 2ibu_1\bar{u}_1u_2 + 2u_0\bar{u}_2u_2 - ibu_1^2\bar{u}_2\\
&\quad - 2ibu_0\bar{u}_1u_3 - ibu_0^2\bar{u}_4,\\
&\;\;\vdots
\end{aligned}$$

$$\begin{aligned}
Q_0 &= -(\lambda+\mu)u_0^2\bar{u}_{0x},\\
Q_1 &= -(\lambda+\mu)\left(u_0^2\bar{u}_{1x} + 2u_0u_1\bar{u}_{0x}\right),\\
Q_2 &= -(\lambda+\mu)\left(u_1^2\bar{u}_{0x} + u_0^2\bar{u}_{2x} + 2u_0u_1\bar{u}_{1x} + 2u_0u_2\bar{u}_{0x}\right),\\
Q_3 &= -(\lambda+\mu)\left(u_1^2\bar{u}_{1x} + u_0^2\bar{u}_{3x} + 2u_0u_1\bar{u}_{2x}\right.\\
&\quad \left.+2u_0u_2\bar{u}_{1x} + 2u_0u_3\bar{u}_{0x} + 2u_1u_2\bar{u}_{0x}\right),\\
Q_4 &= -(\lambda+\mu)\left(u_2^2\bar{u}_{0x} + u_1^2\bar{u}_{2x} + 2u_0u_1\bar{u}_{3x} + 2u_0u_2\bar{u}_{2x}\right.\\
&\quad \left.+2u_0u_3\bar{u}_{1x} + 2u_0u_4\bar{u}_{0x} + 2u_1u_2\bar{u}_{1x} + 2u_1u_3\bar{u}_{0x}\right),\\
&\;\;\vdots\\
R_0 &= (\sigma - 2\lambda - \mu)u_0\bar{u}_0u_{0x},\\
R_1 &= (\sigma - 2\lambda - \mu)(u_0\bar{u}_0u_{1x} + u_0\bar{u}_1u_{0x} + u_1\bar{u}_0u_{0x}),\\
R_2 &= (\sigma - 2\lambda - \mu)\left(u_0\bar{u}_0u_{2x} + u_0\bar{u}_1u_{1x} + u_0\bar{u}_2u_{0x}\right.\\
&\quad \left.+u_1\bar{u}_0u_{1x} + u_1\bar{u}_1u_{0x} + u_2\bar{u}_0u_{0x}\right),\\
R_3 &= (\sigma - 2\lambda - \mu)\left(u_0\bar{u}_0u_{3x} + u_0\bar{u}_1u_{2x} + u_0\bar{u}_2u_{1x}\right.\\
&\quad + u_0\bar{u}_3u_{0x} + u_1\bar{u}_0u_{2x} + u_1\bar{u}_1u_{1x} + u_1\bar{u}_2u_{0x}\\
&\quad \left.+u_2\bar{u}_0u_{1x} + u_2\bar{u}_1u_{0x} + u_3\bar{u}_0u_{0x}\right),\\
R_4 &= (\sigma - 2\lambda - \mu)\left(u_0\bar{u}_0u_{4x} + u_0\bar{u}_1u_{3x} + u_0\bar{u}_2u_{2x}\right.\\
&\quad + u_0\bar{u}_3u_{1x} + u_0\bar{u}_4u_{0x} + u_1\bar{u}_0u_{3x} + u_1\bar{u}_1u_{2x}\\
&\quad + u_1\bar{u}_2u_{1x} + u_1\bar{u}_3u_{0x} + u_2\bar{u}_0u_{2x} + u_2\bar{u}_1u_{1x}\\
&\quad \left.+u_2\bar{u}_2u_{0x} + u_3\bar{u}_0u_{1x} + u_3\bar{u}_1u_{0x} + u_4\bar{u}_0u_{0x}\right),\\
&\;\;\vdots
\end{aligned}$$

Table 2 Dark optical solitons

Cases	a_1	a_2	b	σ	α	λ	μ	ν	κ	B	N	|Max Error|
5	2.00	0.25	1.00	−1.00	−2.00	1.00	−2.00	−2.33	1.00	1.68	12	2.50×10^{-10}
6	1.50	0.20	−3.00	2.00	−1.00	0.33	0.50	−4.71	1.50	3.12	12	2.50×10^{-10}
7	0.20	0.33	−4.00	2.00	2.00	2.00	−2.00	−7.40	2.00	1.15	12	1.00×10^{-10}
8	3.00	0.16	−4.00	3.50	1.00	0.50	1.00	−10.04	1.20	4.83	12	1.50×10^{-10}

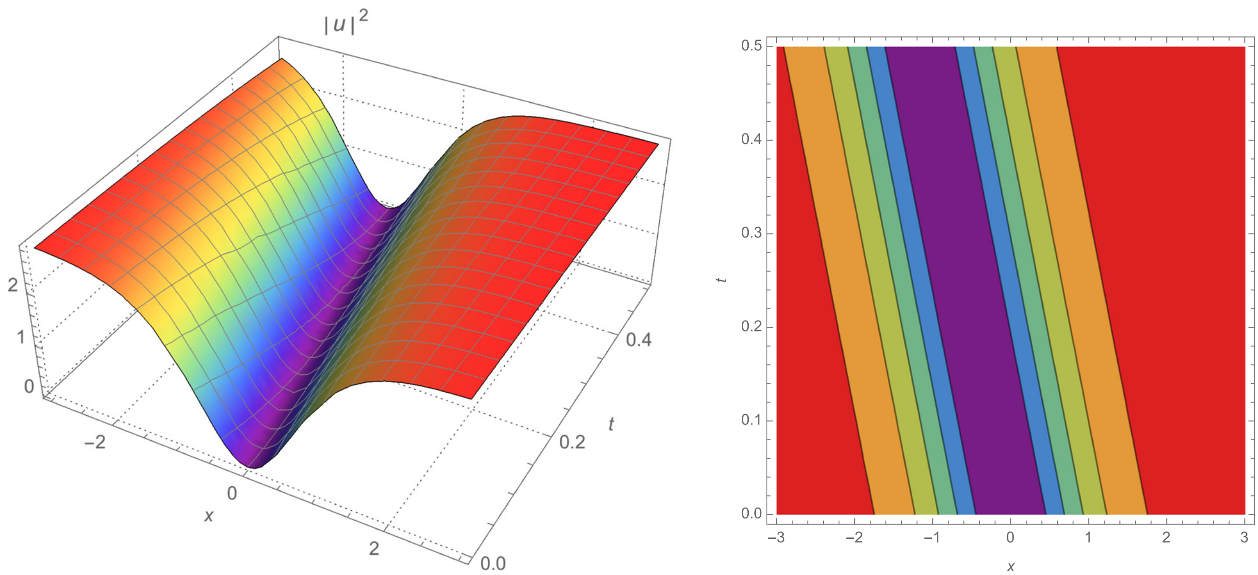

Fig. 5 Dynamic evolution profile of $|u|^2$ via LADM (left) and contour plot of the wave amplitude of $|u|^2$ (right) for the values of the parameters used in case 5 with |Max Error| = 2.5×10^{-10}

Then, the Adomian polynomials corresponding to the nonlinear part $Nu = N_1u + N_2u + N_3u$ are

$$\begin{aligned}
A_0 &= -ibu_0^2\bar{u}_0 - (\lambda+\mu)u_0^2\bar{u}_{0x} + (\sigma - 2\lambda - \mu)u_0\bar{u}_0u_{0x},\\
A_1 &= -2ibu_0u_1\bar{u}_0 - ibu_0^2\bar{u}_1 - (\lambda+\mu)(u_0^2\bar{u}_{1x} + 2u_0u_1\bar{u}_{0x})\\
&\quad + (\sigma - 2\lambda - \mu)(u_0\bar{u}_0u_{1x} + u_0\bar{u}_1u_{0x} + u_1\bar{u}_0u_{0x}),\\
A_2 &= -2ibu_0u_2\bar{u}_0 - ibu_1^2\bar{u}_0 - 2ibu_0u_1\bar{u}_1 - ibu_0^2\bar{u}_2\\
&\quad - (\lambda+\mu)\left(u_1^2\bar{u}_{0x} + u_0^2\bar{u}_{2x} + 2u_0u_1\bar{u}_{1x} + 2u_0u_2\bar{u}_{0x}\right)\\
&\quad + (\sigma - 2\lambda - \mu)(u_0\bar{u}_0u_{2x} + u_0\bar{u}_1u_{1x} + u_0\bar{u}_2u_{0x}\\
&\quad + u_1\bar{u}_0u_{1x} + u_1\bar{u}_1u_{0x} + u_2\bar{u}_0u_{0x}),\\
A_3 &= -2ibu_0u_3\bar{u}_0 - 2ibu_1u_2\bar{u}_0 - 2ibu_0u_2\bar{u}_1 - ibu_1^2\bar{u}_1\\
&\quad - 2ibu_0u_1\bar{u}_2 - ibu_0^2\bar{u}_3 - (\lambda+\mu)(u_1^2\bar{u}_{1x} + u_0^2\bar{u}_{3x}\\
&\quad + 2u_0u_1\bar{u}_{2x} + 2u_0u_2\bar{u}_{1x} + 2u_0u_3\bar{u}_{0x} + 2u_1u_2\bar{u}_{0x})\\
&\quad + (\sigma - 2\lambda - \mu) \times (u_0\bar{u}_0u_{3x} + u_0\bar{u}_1u_{2x} + u_0\bar{u}_2u_{1x}\\
&\quad + u_0\bar{u}_3u_{0x} + u_1\bar{u}_0u_{2x} + u_1\bar{u}_1u_{1x} + u_1\bar{u}_2u_{0x}\\
&\quad + u_2\bar{u}_0u_{1x} + u_2\bar{u}_1u_{0x} + u_3\bar{u}_0u_{0x}),
\end{aligned}$$

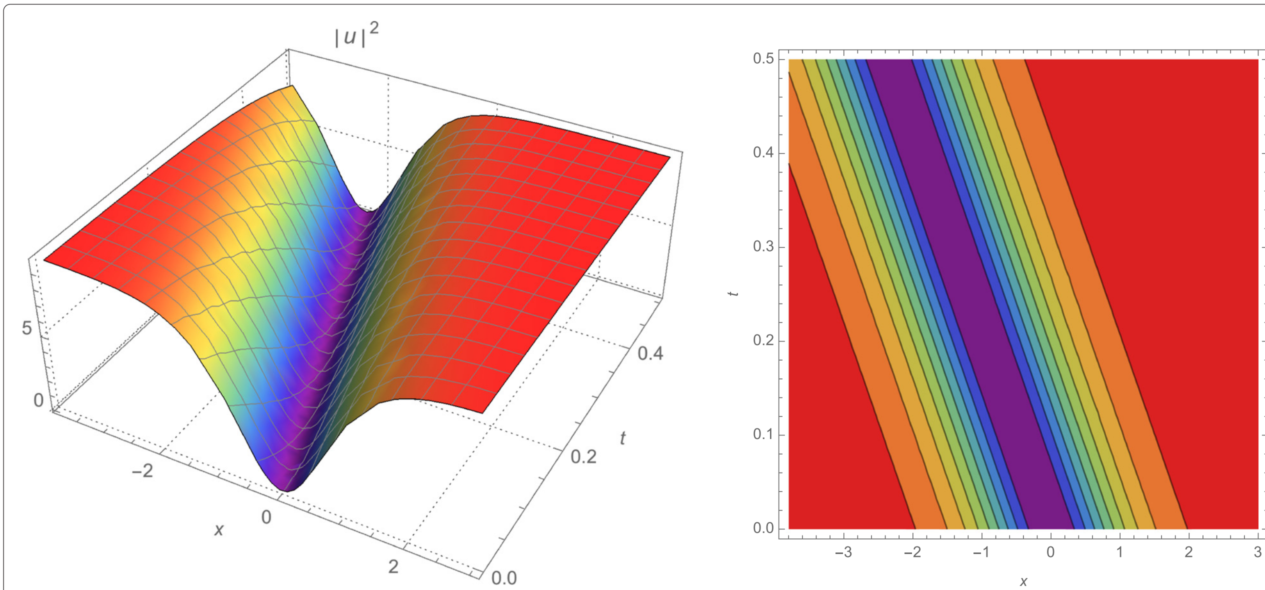

Fig. 6 Dynamic evolution profile of $|u|^2$ via LADM (left) and contour plot of the wave amplitude of $|u|^2$ (right) for the values of the parameters used in case 6 with |Max Error| = 2.5×10^{-10}

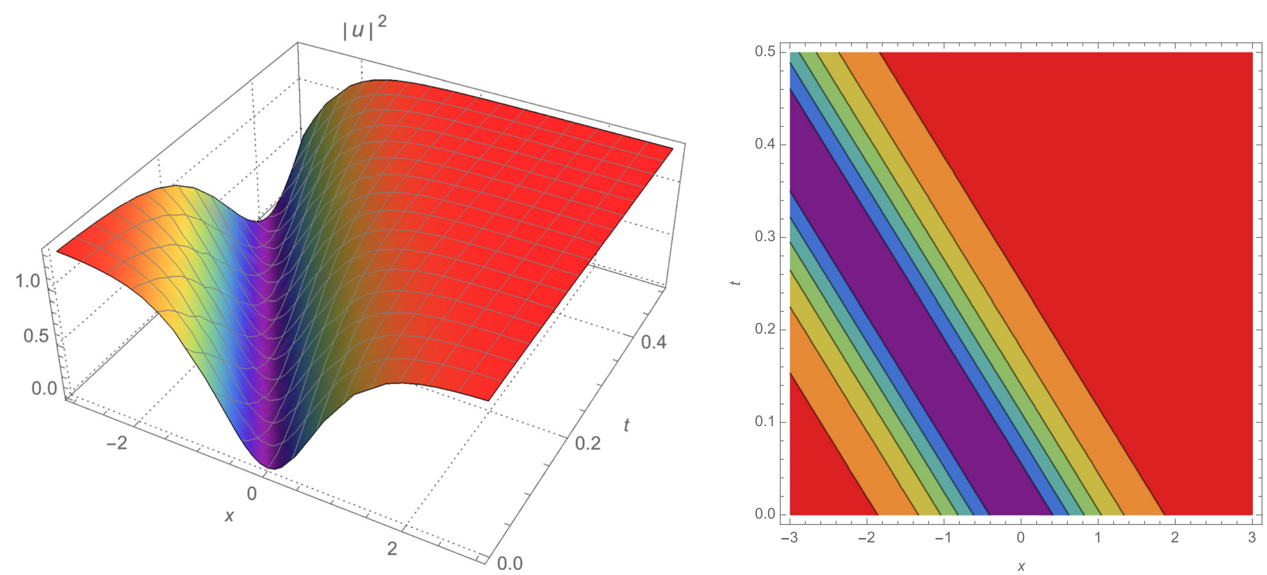

Fig. 7 Dynamic evolution profile of $|u|^2$ via LADM (left) and contour plot of the wave amplitude of $|u|^2$ (right) for the values of the parameters used in case 7 with $|\text{Max Error}| = 1.0 \times 10^{-10}$

and so on for other Adomian polynomials.

By applying the Laplace transform with respect to t on both sides of the Eq. (23) and using the linearity of the Laplace transform gives:

$$\begin{aligned}\mathcal{L}\{Lu(x,t)\} = &-\mathcal{L}\{Ru(x,t)\} - \mathcal{L}\{N_1u(x,t)\}\\ &- \mathcal{L}\{N_2u(x,t)\} - \mathcal{L}\{N_3u(x,t)\}.\end{aligned} \tag{31}$$

Because of the differentiation property of Laplace transform, Eq. (31) can be written as

$$\begin{aligned}s\mathcal{L}\{u(x,t)\} - u(x,0) = &-\mathcal{L}\{Ru(x,t)\} - \mathcal{L}\{N_1u(x,t)\}\\ &- \mathcal{L}\{N_2u(x,t)\} - \mathcal{L}\{N_3u(x,t)\}.\end{aligned} \tag{32}$$

Thus,

$$\begin{aligned}\mathcal{L}\{u(x,t)\} = \frac{1}{s}u(x,0) - \frac{1}{s}\big(&\mathcal{L}\{Ru(x,t)\} + \mathcal{L}\{N_1u(x,t)\}\\ &+\mathcal{L}\{N_2u(x,t)\} + \mathcal{L}\{N_3u(x,t)\}\big).\end{aligned} \tag{33}$$

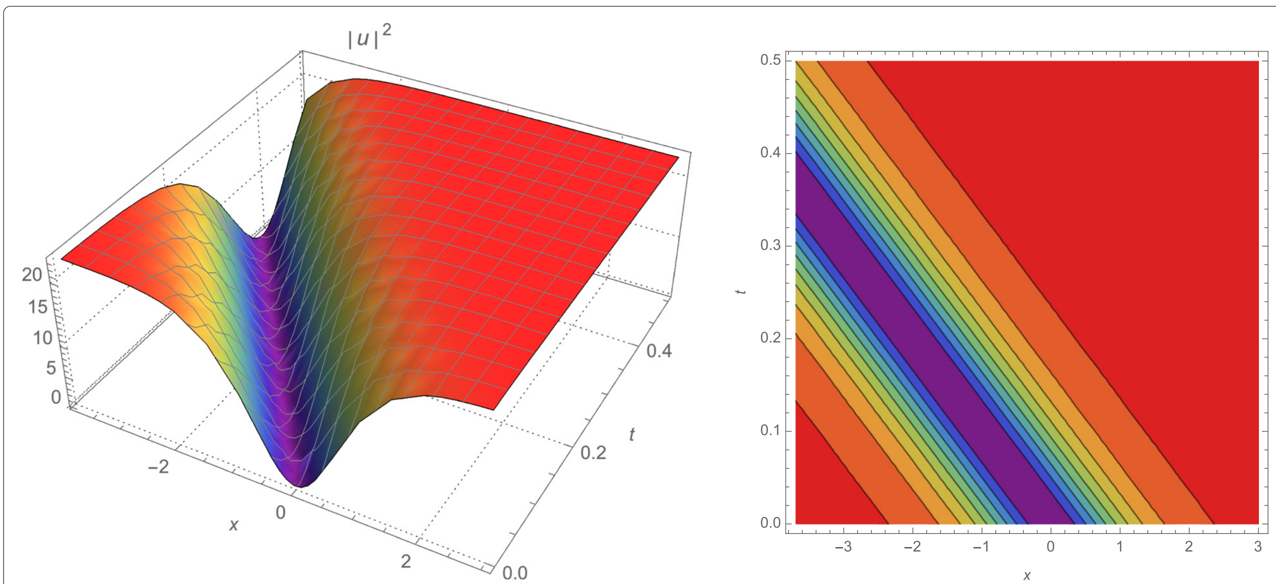

Fig. 8 Dynamic evolution profile of $|u|^2$ via LADM (left) and contour plot of the wave amplitude of $|u|^2$ (right) for the values of the parameters used in case 8 with $|\text{Max Error}| = 1.5 \times 10^{-10}$

By substituting (24), (25), (26) and (27) into (33), we obtain

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} u_n(x,t)\right\} = \frac{f(x)}{s} - \frac{1}{s}\left(\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_n(x,t)\right\} + \mathcal{L}\left\{\sum_{n=0}^{\infty} P_n\right\} + \mathcal{L}\left\{\sum_{n=0}^{\infty} Q_n\right\} + \mathcal{L}\left\{\sum_{n=0}^{\infty} R_n\right\}\right). \tag{34}$$

Comparing both sides of the Eq. (34), the following relations arise:

$$\mathcal{L}\{u_0(x,t)\} = \frac{f(x)}{s} \tag{35}$$

$$\mathcal{L}\{u_1(x,t)\} = -\frac{1}{s}\left(\mathcal{L}\{Ru_0(x,t)\} + \mathcal{L}\{P_0\} + \mathcal{L}\{Q_0\} + \mathcal{L}\{R_0\}\right) \tag{36}$$

$$\mathcal{L}\{u_2(x,t)\} = -\frac{1}{s}\left(\mathcal{L}\{Ru_1(x,t)\} + \mathcal{L}\{P_1\} + \mathcal{L}\{Q_1\} + \mathcal{L}\{R_1\}\right). \tag{37}$$

In general, we get the following recursive algorithm

$$\mathcal{L}\{u_{n+1}(x,t)\} = -\frac{1}{s}\left(\mathcal{L}\{Ru_n(x,t)\} + \mathcal{L}\{P_n\} + \mathcal{L}\{Q_n\} + \mathcal{L}\{R_n\}\right), \quad n \geq 1. \tag{38}$$

Finally, by applying inverse Laplace transformation we deduce the following recurrence formulas for each $n = 0, 1, 2, \ldots,$

$$\begin{cases} u_0(x,t) = f(x), \\ u_{n+1}(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\{Ru_n(x,t) + P_n(u_0,\ldots,u_n) + Q_n(u_0,\ldots,u_n) + R_n(u_0,\ldots,u_n)\}\right]. \end{cases} \tag{39}$$

Numerical simulations and graphical results

We perform numerical simulations for bright and dark optical solitions.

Application to bright optical solitions

The result and the profile of four cases are shown in Table 1 and in Figs. 1, 2, 3 and 4.

Application to dark optical solitions

The result and the profile of four cases are shown in Table 2 and in Figs. 5, 6, 7 and 8.

Conclusions

This paper successfully studied FLE in polarization-preserving fibers by the aid of Laplace-Adomian decomposition scheme. The numerical scheme yielded bright and dark soliton solutions. The results thus appear with a complete spectrum of soliton solutions. Although singular solitons is a third form of solitons that emerge from this model, it does not provide any interest with any kind of numerical scheme. The results of the paper are truly encouraging to study the methodology further along. Later, this scheme will be applied to vector coupled FLE that studies solitons in birefringent fibers. Further along the model will be extended to address WDM/DWDM/UDWDM topology numerically. Such studies are currently under way.

Abbreviations

DWDM: Dense wavelength division multiplexing; FLE: Fokas-lennels equation; GVD: Group velocity dispersion; IMD: Inter-modal dispersion; LADM: Laplace-adomian decomposition method; ND: Nonlinear dispersion; STD: Spatio-temporal dispersion; UDWDM: Ultra-dense wavelength division multiplexing; WDM: Wavelength-division multiplexing

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Authors' contributions

The original ideas and results emerged from discussions among all the authors. OGG wrote the manuscript with input from all authors. All authors read and approved the final manuscript.

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Competing interests

The authors declare that they have no competing interests.

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