Physics Letters A 384 (2020) 126528

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Quasi–monochromatic dynamics of optical solitons having quadratic–cubic nonlinearity

Anjan Biswas [a,b,c,d,*]

[a] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762–4900, USA
[b] Department of Mathematics, King Abdulaziz University, Jeddah–21589, Saudi Arabia
[c] Department of Applied Mathematics, National Research Nuclear University, 31 Kashirskoe Shosse, Moscow–115409, Russian Federation
[d] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria–0008, South Africa

ARTICLE INFO

Article history:
Received 22 March 2020
Received in revised form 27 April 2020
Accepted 28 April 2020
Available online 6 May 2020
Communicated by B. Malomed

Keywords:
Quasimonochromaticity
Soliton perturbation
Quadratic–cubic law

ABSTRACT

This paper obtains the adiabatic parameter dynamics and illustrates optical soliton cooling with having quadratic–cubic nonlinearity. The soliton perturbation theory is implemented to picture the dynamical system.

1. Introduction

Optical solitons form the basic fabric in telecommunications industry. These soliton molecules transmit loads of information across the globe in a matter of a few femto–seconds. The speed and efficiency of soliton dynamics is very commendable and therefore is the current cutting edge technology of telecommunications industry. There is a wide variety of aspects of soliton science in optics, Bose–Einstein condensates that are addressed in a wide variety of works [1–15]. These include conservation laws, collision–induced timing jitter, stochastic perturbation, ghost pulses evolution, cross-talk and many others. This paper studies a very important aspect of soliton dynamics that is addressed with quadratic–cubic (QC) law of refractive index.

One of the most important features in optical soliton dynamics is the aspect of quasi–monochromaticity. The departure from the carrier frequency of the soliton leads to relative width of the spectrum [5–7,9]. This leads to an abundance of perturbation terms and hence the soliton perturbation theory comes to play that leads to the adiabatic parameter dynamics of such solitons. This dynamics is applicable for both bright as well as dark solitons. The current work focuses on obtaining this adiabatic parameter dynamics of bright solitons only that is studied with QC law of refractive index [1–4,14]. The adiabatic dynamics of soliton power and soliton frequency are recovered that led to its stable fixed point which eventually yielded the effect of optical soliton cooling. The velocity of the soliton also undergoes small change and its dynamics has also been captured for the model. The perturbation terms are of Hamiltonian as well as non-Hamiltonian type and finally a couple of non–local type perturbation effects are also addressed. The details are sketched in the rest of the paper after a quick re-visitation of the unperturbed model [4,14].

1.1. Governing model

The dimensionless form of the governing nonlinear Schrödinger's equation with QC nonlinearity is written as [4,14]:

$$iq_t + aq_{xx} + \left(b_1 |q| + b_2 |q|^2\right) q = 0 \tag{1}$$

* Correspondence to: Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762–4900, USA.
E-mail address: biswas.anjan@gmail.com.

https://doi.org/10.1016/j.physleta.2020.126528
0375-9601/

where $q(x,t)$ is a complex–valued dependent variable that represents the wave profile and $i=\sqrt{-1}$. The independent variables x and t are the spatial and temporal variables respectively. The constants a and b_j for $j=1,2$ are real–valued and represent chromatic dispersion and self–phase modulation respectively. This model had been extensively studied, both in polarization–preserving fibers as well as birefringent fibers, and soliton solutions have been recovered [1–4,14]. The bright 1–soliton solution is given by [14]:

$$q(x,t)=\frac{A}{D+\cosh[B(x-vt)]}e^{i(-\kappa x+\omega t+\theta_0)} \tag{2}$$

where A represents the soliton amplitude, B is its width with a parameter is D. The phase component gives κ as the soliton frequency and ω is the soliton wave number while θ_0 is the phase constant. The velocity of the soliton is related as:

$$v=-2a\kappa. \tag{3}$$

Finally, the amplitude (A), the width (B) and the parameter D are respectively given as:

$$A=\frac{6\left(\omega+a\kappa^2\right)}{\sqrt{4b_1^2+18b_2\left(\omega+a\kappa^2\right)}}, \tag{4}$$

$$B=\sqrt{\frac{\omega+a\kappa^2}{a}}, \tag{5}$$

and

$$D=\frac{2b_1}{\sqrt{4b_1^2+18b_2\left(\omega+a\kappa^2\right)}}. \tag{6}$$

These relations yield the following couple of parameter restrictions:

$$2b_1^2+9\left(\omega+a\kappa^2\right)>0 \tag{7}$$

and

$$a\left(\omega+a\kappa^2\right)>0. \tag{8}$$

The conservation laws for (1) are [14]:

$$P=\int_{-\infty}^{\infty}|q|^2\,dx=\frac{2A^2}{3B}F\left(2,2;\frac{5}{2};\frac{1-D}{2}\right) \tag{9}$$

$$M=ia\int_{-\infty}^{\infty}\left(qq_x^*-q^*q_x\right)dx=\frac{2a\kappa A^2}{3B}F\left(2,2;\frac{5}{2};\frac{1-D}{2}\right) \tag{10}$$

In (9) and (10), the Gauss' hypergeometric function is written as:

$$F\left(\alpha,\beta;\gamma;z\right)=\sum_{n=0}^{\infty}\frac{(\alpha)_n\,(\beta)_n}{(\gamma)_n}\frac{z^n}{n!} \tag{11}$$

where the Pochhammer symbol is:

$$(p)_n=\begin{cases}1 & n=0,\\ p(p+1)\cdots(p+n-1) & n>0.\end{cases} \tag{12}$$

The condition that guarantees convergence of the series is casted as

$$|z|<1, \tag{13}$$

which for (9) and (10) means

$$-1<D<3. \tag{14}$$

Finally, Rabbe's test of convergence reveals another criterion for convergence of the series, namely

$$\gamma<\alpha+\beta \tag{15}$$

which is valid for both of the hypergeometric functions listed in (9) or (10).

2. Soliton perturbation

The perturbed NLSE with QC nonlinearity is written as [5–7,9]

$$iq_t + aq_{xx} + \left(b_1 |q| + b_2 |q|^2\right) q = i\epsilon R \tag{16}$$

where R represents perturbation terms while the perturbation parameter ϵ is from quasi–monochromaticity and represents the relative width of the spectrum with typically $0 < \epsilon \ll 1$. When the perturbation terms are turned on, the adiabatic variation of the soliton power and frequency are governed as [5–7,9]:

$$\frac{dP}{dt} = \epsilon \int_{-\infty}^{\infty} \left(q^* R + qR^*\right) dx \tag{17}$$

and

$$\frac{d\kappa}{dt} = \frac{\epsilon}{2P}\left[i\left(q_x^* R - q_x R^*\right) dx - a\kappa \int_{-\infty}^{\infty} \left(q^* R + qR^*\right) dx \right] \tag{18}$$

The variation of the soliton velocity is also governed by the relation:

$$v = -2a\kappa - \frac{\epsilon}{P} \int_{-\infty}^{\infty} x\left(q^* R + qR^*\right) dx \tag{19}$$

2.1. Adiabatic parameter dynamics

The perturbation terms studied in this paper are written as [7,9]:

$$\begin{aligned} R = {} & \delta |q|^{2m} q + \alpha q_x + \beta q_{xx} - \gamma q_{xxx} + \lambda \left(|q|^2 q\right)_x + \theta \left(|q|^2\right)_x q + \rho |q_x|^2 q \\ & - i\xi \left(q^2 q_x^*\right)_x - i\eta q_x^2 q^* - i\zeta q^* \left(q^2\right)_{xx} - i\mu \left(|q|^2\right)_x q + (\sigma_1 q + \sigma_2 q_x) \int_{-\infty}^{x} |q|^2 ds \end{aligned} \tag{20}$$

Here, in (20), δ represents multi–photon absorption, while α is the inter–modal dispersion, β represents band–pass filters while γ is the third–order dispersion. Next, λ is the self–steepening effect, while θ gives the effect of nonlinear dispersion and ρ accounts for nonlinear dissipation. Then again, ξ, η and ζ stem from quasilinear pulses while μ is from intra–pulse Raman scattering and finally, σ_j for $j = 1, 2$ gives saturable nonlinear effect.

With these perturbation terms, the adiabatic variation of the soliton power and frequency are indicated below:

$$\begin{aligned} \frac{dP}{dt} = {} & \epsilon \Bigg[\frac{\delta A^{2m+2}}{2^{2m} B} \frac{8m(2m+1)}{(4m+1)(4m+3)} F\left(2m+2, 2m+2; 2m+\frac{5}{2}; \frac{1-D}{2}\right) \frac{\Gamma(2m)\,\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(2m+\frac{1}{2}\right)} \\ & - \frac{4\beta A^2}{3B}\left\{ B^2 F\left(4, 2; \frac{7}{2}; \frac{1-D}{2}\right) + \kappa^2 F\left(2, 2; \frac{5}{2}; \frac{1-D}{2}\right)\right\} \\ & + \frac{4\rho A^4}{105B}\left\{ 7B^2 F\left(4, 2; \frac{7}{2}; \frac{1-D}{2}\right) + 3\kappa^2 F\left(4, 4; \frac{9}{2}; \frac{1-D}{2}\right)\right\} \\ & + \frac{2\sigma_1 A^4}{B^2} \int_{-\infty}^{\infty} \frac{1}{(D+\cosh[B(x-vt)])^2} \left\{ \frac{2}{\left(1-D^2\right)^{\frac{3}{2}}} \tan^{-1}\left[\frac{(D-1)}{\sqrt{1-D^2}} \tanh\frac{x}{2}\right] + \frac{2D}{\left(1-D^2\right)^{\frac{3}{2}}} \tan^{-1}\left(\frac{D-1}{\sqrt{1-D^2}}\right) \right. \\ & \left. - \frac{1}{\left(D^2-1\right)} \frac{\sinh x}{D+\cosh x} \right\} dx \\ & - \frac{2\sigma_2 A^4}{B} \int_{-\infty}^{\infty} \frac{\sinh[B(x-vt)]}{(D+\cosh[B(x-vt)])^3} \left\{ \frac{2}{\left(1-D^2\right)^{\frac{3}{2}}} \tan^{-1}\left[\frac{(D-1)}{\sqrt{1-D^2}} \tanh\frac{x}{2}\right] + \frac{2D}{\left(1-D^2\right)^{\frac{3}{2}}} \tan^{-1}\left(\frac{D-1}{\sqrt{1-D^2}}\right) \right. \\ & \left. - \frac{1}{\left(D^2-1\right)} \frac{\sinh x}{D+\cosh x} \right\} dx \Bigg] \end{aligned} \tag{21}$$

$$\frac{d\kappa}{dt} = \epsilon\kappa \left[\frac{4\kappa A^2}{3aP} F\left(2, 2; \frac{5}{2}; \frac{1-D}{2}\right) - \frac{16\mu A^4 B}{315P} F\left(6, 4; \frac{11}{2}; \frac{1-D}{2}\right) \right.$$

$$
\begin{aligned}
&-\frac{2\sigma_1(1-a)A^4}{PB^2}\int_{-\infty}^{\infty}\frac{1}{(D+\cosh[B(x-vt)])^2}\left\{\frac{2}{(1-D^2)^{\frac{3}{2}}}\tan^{-1}\left[\frac{(D-1)}{\sqrt{1-D^2}}\tanh\frac{x}{2}\right]+\frac{2D}{(1-D^2)^{\frac{3}{2}}}\tan^{-1}\left(\frac{D-1}{\sqrt{1-D^2}}\right)\right.\\
&\left.-\frac{1}{(D^2-1)}\frac{\sinh x}{D+\cosh x}\right\}dx\\
&+\frac{2\sigma_2 aA^4}{PB}\int_{-\infty}^{\infty}\frac{\sinh[B(x-vt)]}{(D+\cosh B[x-vt)])^2}\left\{\frac{2}{(1-D^2)^{\frac{3}{2}}}\tan^{-1}\left[\frac{(D-1)}{\sqrt{1-D^2}}\tanh\frac{x}{2}\right]+\frac{2D}{(1-D^2)^{\frac{3}{2}}}\tan^{-1}\left(\frac{D-1}{\sqrt{1-D^2}}\right)\right.\\
&\left.\left.-\frac{1}{(D^2-1)}\frac{\sinh x}{D+\cosh x}\right\}dx\right]
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
v=-2a\kappa+\epsilon&\left[\frac{2\alpha A^2}{3Bp}F\left(2,2;\frac{5}{2};\frac{1-D}{2}\right)+\frac{2\gamma A^2}{5BP}\left\{B^2F\left(4,2;\frac{7}{2};\frac{1-D}{2}\right)+5\kappa^2F\left(2,2;\frac{5}{2};\frac{1-D}{2}\right)\right\}\right.\\
&+\frac{4\kappa\left(\xi-\eta+4\zeta\right)A^4}{35BP}F\left(4,4;\frac{9}{2};\frac{1-D}{2}\right)\\
&\left.++\frac{2\sigma_1A^4}{PB^2\sqrt{1-D^2}}\int_{-\infty}^{\infty}\left\{\tan^{-1}\left[\frac{(D-1)}{\sqrt{1-D^2}}\tanh\frac{x}{2}\right]+\tan^{-1}\left(\frac{D-1}{\sqrt{1-D^2}}\right)\right\}^2dx\right]
\end{aligned}
\tag{23}
$$

The dynamical system given by (21)–(22) has a fixed point $(\bar{A},\bar{\kappa})=(\bar{A},0)$, namely a *sink*. Here the value of the amplitude $\bar{A}$ is retrievable from the implicit equation

$$
\begin{aligned}
&\frac{\delta A^{2m}}{2^{2m}}\frac{8m(2m+1)}{(4m+1)(4m+3)}F\left(2m+2,2m+2;2m+\frac{5}{2};\frac{1-D}{2}\right)\frac{\Gamma(2m)\,\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(2m+\frac{1}{2}\right)}-\frac{4\beta B^2}{3}F\left(4,2;\frac{7}{2};\frac{1-D}{2}\right)\\
&+\frac{2\sigma_1A^2}{B}\int_{-\infty}^{\infty}\frac{1}{(D+\cosh[B(x-vt)])^2}\left\{\frac{2}{(1-D^2)^{\frac{3}{2}}}\tan^{-1}\left[\frac{(D-1)}{\sqrt{1-D^2}}\tanh\frac{x}{2}\right]+\frac{2D}{(1-D^2)^{\frac{3}{2}}}\tan^{-1}\left(\frac{D-1}{\sqrt{1-D^2}}\right)\right.\\
&\left.-\frac{1}{(D^2-1)}\frac{\sinh x}{D+\cosh x}\right\}dx\\
&-2\sigma_2A^2B^2\int_{-\infty}^{\infty}\frac{\sinh[B(x-vt)]}{(D+\cosh[B(x-vt)])^3}\left\{\frac{2}{(1-D^2)^{\frac{3}{2}}}\tan^{-1}\left[\frac{(D-1)}{\sqrt{1-D^2}}\tanh\frac{x}{2}\right]+\frac{2D}{(1-D^2)^{\frac{3}{2}}}\tan^{-1}\left(\frac{D-1}{\sqrt{1-D^2}}\right)\right.\\
&\left.-\frac{1}{(D^2-1)}\frac{\sinh x}{D+\cosh x}\right\}dx=0
\end{aligned}
\tag{24}
$$

Thus, the perturbed soliton will travel down the optical fiber, having QC nonlinearity, with a fixed amplitude $\bar{A}$ and maintain the speed of light. Thus, the phenomena of *optical soliton cooling* is illustrated with QC nonlinearity.

Here, the constraint condition (14), by virtue of (21)–(24), transforms to:

$$
-1<\frac{2b_1}{\sqrt{4b_1^2+18b_2\left(\omega+a\kappa^2\right)}}<1.
\tag{25}
$$

Fig. 1 represents the profile of perturbed bright 1–soliton solution to the model given by (17). The parameter values chosen are $\epsilon=0.05$, $a=b_1=b_2=1$, $\kappa=-\omega=-0.2$, $\alpha=\beta=0.1$, $\lambda=0.2$, $\gamma=1$ and $\rho=\xi=0.1$.

3. Conclusions

This paper laid down the adiabatic parameter dynamics of optical solitons from QC nonlinearity. The phenomena of optical soliton cooling is also illustrated. It must be noted that it is not possible to find the evolution of all the parameters, namely the amplitude (A), inverse width (B), velocity (v), frequency (κ) and the wave number (ω) from the two conservation laws. In that case, "variation of parameters", method of "collective variables" as well as "method of moments" become handy [16–19].

The results are thus meaningful and carries a lot of future research prospects. These results lay a strong foundation to study additional aspects with the model, such as four–wave mixing, collision–induced timing jitter, addressing stochastic perturbation [20] and several others. One additional issue is to study this quasi–monochromatic soliton dynamics with dark solitons as well as bright–dark solitons [15]. These studies are all under way and the results are currently awaited.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

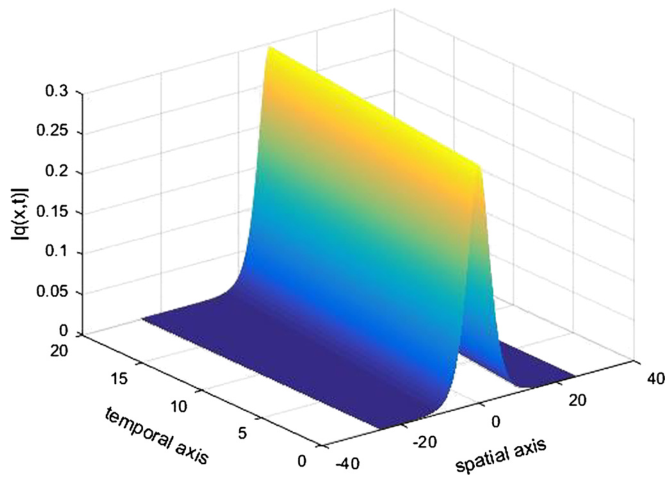

Fig. 1. Perturbed bright 1-soliton solution propagation.

References

[1] M. Asma, W.A.M. Othman, B.R. Wong, A. Biswas, Optical soliton perturbation with quadratic–cubic nonlinearity by semi–inverse variational principle, Proc. Rom. Acad., Ser. A 18 (4) (2017) 331–336.
[2] M. Asma, W.A.M. Othman, B.R. Wong, A. Biswas, Optical soliton perturbation with quadratic–cubic nonlinearity by the method of undetermined coefficients, J. Optoelectron. Adv. Mater. 19 (11–12) (2017) 699–703.
[3] M. Asma, W.A.M. Othman, B.R. Wong, A. Biswas, Optical soliton perturbation with tquadratic–cubic nonlinearity by traveling wave hypothesis, Optoelectron. Adv. Mater., Rapid Commun. 11 (9–10) (2017) 517–519.
[4] G.E. Astrakharchik, B.A. Malomed, Dynamics of one-dimensional quantum droplets, Phys. Rev. A 98 (1) (2018) 013631.
[5] A. Biswas, S. Konar, Introduction to Non-Kerr Law Optical Solitons, CRC Press, Boca Raton, FL. USA, 2006.
[6] A. Biswas, D. Milovic, F. Majid, R. Kohl, Optical soliton cooling in a saturabble law media, J. Electromagn. Waves Appl. 22 (13) (2008) 1735–1746.
[7] A. Biswas, Q. Zhou, M.Z. Ullah, M. Asma, S.P. Moshokoa, M. Belic, Perturbation theory and optical soliton cooling with anti–cubic nonlinearity, Optik 142 (2017) 73–76.
[8] J. Fujioka, E. Cortés, R. Perez–Pascual, R.F. Rodríguez, A. Espinosa, B.A. Malomed, Chaotic solitons in the quadratic–cubic nonlinear Schrödinger equation under nonlinearity management, Chaos 21 (2011) 033120.
[9] L. Girgis, D. Milovic, T. Hayat, O.M. Aldossary, A. Biswas, Optical soliton perturbation with log–law nonlinearity, Opt. Appl. 42 (3) (2012) 447–454.
[10] K. Hayata, M. Koshiba, Prediction of unique solitary–wave polaritons in quadratic–cubic nonlinear dispersive media, J. Opt. Soc. Am. B 11 (12) (1994) 2581–2585.
[11] L. Khaykovich, B.A. Malomed, Deviation from one–dimensionality in stationary properties and collisional dynamics of matter–wave solitons, Phys. Rev. A 74 (2006) 023607.
[12] R.K. Kumar, L. Tomio, B.A. Malomed, A. Gammal, Vortex lattices in binary Bose–Einstein condensates with dipole-dipole interactions, Phys. Rev. A 96 (2017) 063624.
[13] Y. Li, Y. Liu, Z. Fan, W. Pang, S. Fu, B.A. Malomed, Two-dimensional dipolar gap solitons in free space with spin-orbit coupling, Phys. Rev. A 95 (6) (2017) 063613.
[14] H. Triki, A. Biswas, S.P. Moshokoa, M. Belic, Optical solitons and conservation laws with quadratic–cubic nonlinearity, Optik 128 (2017) 63–70.
[15] Y. Qiu, B.A. Malomed, D. Mihalache, X. Zhu, J. Peng, Y. He, Generation of stable multi–vortex clusters in a dissipative medium with anti–cubic nonlinearity, Phys. Lett. A 383 (22) (2019) 2579–2583.
[16] M. Asma, W.A.M. Othman, B.R. Wong, A. Biswas, Chirped optical Gausson perturbation with quadratic–cubic nonlinearity by collective variables, Opt. Quantum Electron. 51 (6) (2019) 200.
[17] A. Hasegawa, Y. Kodama, Solitons in Optical Communications, Oxford University Press, Oxford, UK, 1995.
[18] A. Biswas, Dispersion-managed solitons in optical fibers, J. Opt. A 4 (1) (2002) 84–97.
[19] C. Elloh, G. Edah, A. Ayela, A. Biswas, M. Ekici, M. Asma, A.K. Alzahrani, M.R. Belic, Gausson parameter dynamics in ENZ–material based waveguides using moment method, submitted for publication.
[20] S. Khan, Stochastic perturbation of optical solitons with quadratic–cubic law of refractive index, Optik 212 (2020) 164706.