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Optical Solitons with Kudryashov's Equation by Lie Symmetry Analysis[1]

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Abstract—In this work, Kudryashov's equation is studied with Lie symmetry analysis, which is implemented to describe the propagation pulses in an optical fiber. The equation is converted into system of ordinary differential equations with similarity transformations. These gave way to bright, dark and singular optical soliton solutions to the model.

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1. INTRODUCTION

Optical solitons are aggressively pursued with several models that have existed since the past few decades [1–17]. There are several equations that have been lately proposed to address the dynamics of soliton propagation through a variety of waveguides such as optical fibers, crystals, metamaterials, PCF and magneto-optic waveguides. The most familiar model is the nonlinear Schrödinger's equation [2, 7, 11, 15]. A few other models, governing soliton dynamics, are Biswas–Arshed equation [8, 16], Triki–Biswas equation [1, 10], and the Radhakrishan–Kundu–Laksmanan equation [13] that come with different forms of nonlinear refractive index. Recently, N. Kudryashov proposed a law of refractive index that led to Kudryashov's equation (KE) [9]. KE is used to describe the propagation pulses in an optical fiber. The solutions to KE have been recovered with extended trial function [4], F-expansion [5] and undetermined coefficients [6]. The current paper handles KE by Lie symmetry analysis.

Here, first we generate infinitesimals and Lie symmetries of KE. Then two vector fields are obtained. With the help of these vector fields, the governing equation is reduced into system of ordinary differential equations (ODEs). Then exact optical soliton solutions of system of ODEs are recovered. Finally, the corresponding bright, dark and singular optical soliton solutions of governing equation are presented.

1.1. Governing Model

The dimensionless form of KE is given by [4–6, 9]

$$iq_t + aq_{xx} + \left(b|q|^{2n} + c|q|^{n} + g|q|^{-n} + h|q|^{-2n}\right)q = 0. \quad (1)$$

In Eq. (1), the first term indicates the linear temporal evolution and a indicates the coefficients of chromatic dispersion. The remaining terms are nonlinear and stem from the law of refractive index of an optical fiber and gives self-phase modulation effect to the model. At $n = 1$; by taking $g = h = 0$ and $c = g = h = 0$, the model referred as parabolic law and Kerr law respectively, which has been extensively studied. Other special cases are power law and dual-power laws of nonlinearity that has also been extensively studied. The goal of the current paper will address Eq. (1), as it stands, extensively and exhaustively by Lie symmetry analysis.

2. LIE SYMMETRY ANALYSIS

In this section, we will employ Lie classical method [18–20] on Eq. (1) in order to obtain the infinitesimals. To achieve this goal, let us consider

[1] The text was submitted by the authors in English.

$$q(x,t) = u(x,t)\exp[iv(x,t)], \tag{2}$$

where u and v are real-valued functions. Equation (2) transforms Eq. (1) into imaginary and real portions as

$$\begin{gathered} u_t + 2au_xv_x + auv_{x,x} = 0, \quad au_{x,x} - uv_t - auv_x^2 \\ + (bu^{2n} + cu^n + gu^{-n} + hu^{-2n})u = 0. \end{gathered} \tag{3}$$

For the system of equations (3), let us consider one-parameter (ε) transformations as

$$\begin{aligned} x^* &= x + \varepsilon\xi(x,t,u,v) + O(\varepsilon^2), \\ t^* &= t + \varepsilon\tau(x,t,u,v) + O(\varepsilon^2), \\ u^* &= u + \varepsilon\eta(x,t,u,v) + O(\varepsilon^2), \\ v^* &= v + \varepsilon\phi(x,t,u,v) + O(\varepsilon^2), \end{aligned} \tag{4}$$

where ξ, τ, η, and ϕ are infinitesimals, depending upon x, t, u, v, have to be determined.

The vector field associated with these transformations is

$$V = \xi\partial_x + \tau\partial_t + \eta\partial_u + \phi\partial_v. \tag{5}$$

For system of equations (3), the second prolongations formula [19, 20] are

$$\begin{gathered} pr^{(2)}V = V + \eta^x\frac{\partial}{\partial u_x} + \phi^x\frac{\partial}{\partial v_x} + \eta^t\frac{\partial}{\partial u_t} + \phi^{xx}\frac{\partial}{\partial v_{xx}}, \\ pr^{(2)}V = V + \phi^x\frac{\partial}{\partial v_x} + \phi^t\frac{\partial}{\partial v_t} + \eta^{xx}\frac{\partial}{\partial u_{xx}}, \end{gathered} \tag{6}$$

where η^x, η^t, ϕ^x, ϕ^t, ϕ^{xx}, and η^{xx} are extended infinitesimals.

From the invariance conditions $pr^{(2)}V(\Delta) = 0$ whenever $\Delta = 0$ in(3), we have

$$\begin{gathered} 0 = a\eta v_{xx} + au\phi^{xx} + 2a\left(\eta^x v_x + \phi^x u_x\right) + \eta^t, \\ 0 = a\eta^{xx} - 2auv_x\phi^x - u\phi^t - \left(av_x^2 + v_t\right)\eta + \left[(2n+1)bu^{2n} + (n+1)cu^n - (n-1)gu^{-n} - (2n-1)hu^{-2n}\right]\eta. \end{gathered} \tag{7}$$

Substituting the values of infinitesimals η^x, η^t, ϕ^x, ϕ^t, ϕ^{xx}, and η^{xx}, and by equating the coefficient of various derivative terms equal to zero, we get the system of PDEs. By solving this system, we get

$$\xi = C_2 + tC_1, \quad \tau = C_1, \quad \eta = 0, \quad \phi = C_3 + \frac{x}{2a}C_4, \tag{8}$$

where C_1, C_2, C_3, and C_4 are arbitrary constants. Hence Lie algebra of system (3) is spanned by the infinitesimal generators as

$$V_1 = \frac{\partial}{\partial t}, \quad V_2 = \frac{\partial}{\partial x}, \quad V_3 = \frac{\partial}{\partial v}, \quad V_4 = t\frac{\partial}{\partial x} + \frac{x}{2a}\frac{\partial}{\partial v}. \tag{9}$$

The commutation relations determined of V_1, V_2, V_3, and V_4 are given by

$$\begin{gathered} [V_1,V_2] = 0, \quad [V_1,V_3] = 0, \quad [V_1,V_4] = V_2, \\ [V_2,V_3] = 0, \quad [V_2,V_4] = \frac{V_3}{2a}, \quad [V_1,V_4] = 0. \end{gathered} \tag{10}$$

3. SYMMETRY REDUCTION AND INVARIANT SOLUTIONS

To derive invariant solutions of system (3), we have to solve the corresponding characteristic equation given as

$$\frac{dx}{\xi} = \frac{dt}{\tau} = \frac{du}{\eta} = \frac{dv}{\phi}, \tag{11}$$

where ξ, τ, η, and ϕ are presented by (8). To solve characteristic Eq. (11), we will consider two cases of vector fields: (i) V_4 and (ii) $V_3 + \beta V_2 + \mu V_1$, where β and μ are arbitrary non-zero real numbers.

3.1. Case (i): V_4

By solving characteristic Eq. (11), we have following similarity variables:

$$s = t, \quad u(x,t) = P(s), \quad v(x,t) = \frac{x^2}{4at} + Q(s), \tag{12}$$

where P and Q are new dependent variables, which depends upon s.

Using (12) in (3), we have

$$P' + \frac{1}{2s}P = 0, \tag{13}$$

$$Q' - \left(bP^{2n} + cP^n + gP^{-n} + hP^{-2n}\right) = 0, \tag{14}$$

where prime represents derivative w.r.t. s. From (13), we have

$$P(s) = k_1/\sqrt{s}, \tag{15}$$

where k_1 is arbitrary constant.

By substituting (15) into (14) and solving, we have following cases:

(a) When $n = -2$

$$Q(s) = \frac{1}{3}\frac{bs^3}{k_1^4} + \frac{1}{2}\frac{cs^2}{k_1^2} + gk_1^2\ln s - \frac{hk_1^4}{s} + k_2. \tag{16}$$

(b) When $n = -1$

$$Q(s) = \frac{1}{2}\frac{bs^2}{k_1^2} + \frac{2}{3}\frac{cs^{3/2}}{k_1} + 2gk_1\sqrt{s} + hk_1^2\ln s + k_3. \tag{17}$$

(c) When $n = 1$

$$Q(s) = bk_1^2 \ln s + 2ck_1\sqrt{s} + \frac{2}{3}\frac{gs^{3/2}}{k_1} + \frac{1}{2}\frac{hs^2}{k_1^2} + k_4. \quad (18)$$

(d) When $n = 2$

$$Q(s) = --\frac{bk_1^4}{s} + ck_1^2 \ln s + \frac{1}{2}\frac{gs^2}{k_1^2} + \frac{1}{3}\frac{hs^3}{k_1^4} + k_5. \quad (19)$$

(e) When $n \neq -2, -1, 1$, and 2

$$\begin{aligned} Q(s) = &-\frac{bk_1^{2n}}{n-1}s^{1-n} - \frac{2ck_1^n}{n-2}s^{1-n/2} \\ &+ \frac{2gk_1^{-n}}{n+2}s^{1+n/2} + \frac{hk_1^{-2n}}{n+1}s^{1+n} + k_6, \end{aligned} \quad (20)$$

where k_2, k_3, k_4, k_5, and k_6, are arbitrary constants. Hence, corresponding solution of (1) is given by

$$q(x,t) = \frac{k_1}{\sqrt{t}}\exp\left[i\left(\frac{x^2}{4at} + Q\right)\right], \quad (21)$$

where Q is given by (16)–(20) and s is given by (12).

3.2. Case (ii): $V_3 + \beta V_2 + \mu V_1$

By solving characteristic Eq. (11), we have following similarity variables:

$$\rho = \mu x - \beta t, \quad u(x,t) = M(\rho), \quad v(x,t) = \frac{x}{\beta} + N(\rho), \quad (22)$$

where M and N are new dependent variables, which depends upon ρ.

Substituting (22) in (3), we have

$$\left(2a\mu - \beta^2 + 2a\beta\mu^2 N'\right)M' + a\beta\mu^2 MN'' = 0, \quad (23)$$

$$\begin{aligned} &a\beta^2\mu^2 M'' + \left[\left(\beta^3 - 2a\mu\beta\right)N' - a - a\beta^2\mu^2 N'^2\right]M \\ &+ \beta^2\left(bM^{2n} + cM^n + gM^{-n} + hM^{-2n}\right)M = 0, \end{aligned} \quad (24)$$

where prime represents derivative w.r.t. ρ. Double integral of (23) gives

$$N = \frac{\beta^2 - 2a\mu}{2a\beta\mu^2}\rho + \frac{k_7}{2a\beta\mu^2}\int\frac{1}{M^2}d\rho + k_8, \quad (25)$$

where k_7 and k_8 are arbitrary constants.

Substituting (25) into (24), we have

$$\begin{aligned} M'' = &\frac{1}{4}\frac{4a\mu - \beta^2}{a^2\mu^4}M + \frac{1}{4}\frac{k_7^2}{a^2\beta^2\mu^4}M^{-3} \\ &- \frac{1}{a\mu^2}\left(bM^{1+2n} + cM^{1+n} + gM^{1-n} + hM^{1-2n}\right). \end{aligned} \quad (26)$$

(a) When $n = -2$, Eq. (26) can be written as

$$\begin{aligned} M'' = &\frac{1}{4}\frac{4a\mu - \beta^2}{a^2\mu^4}M + \frac{1}{4}\frac{k_7^2}{a^2\beta^2\mu^4}M^{-3} \\ &- \frac{1}{a\mu^2}\left(bM^{-3} + cM^{-1} + gM^3 + hM^5\right). \end{aligned} \quad (27)$$

Integrating (27), we have

$$\begin{aligned} \frac{(M')^2}{2} = &\frac{1}{8}\frac{4a\mu - \beta^2}{a^2\mu^4}M^2 - \frac{1}{8}\frac{k_7^2}{a^2\beta^2\mu^4}M^{-2} \\ &- \frac{1}{a\mu^2}\left(-\frac{b}{2}M^{-2} + c\ln M + \frac{g}{4}M^4 + \frac{h}{6}M^6\right) + \frac{k_9}{8}, \end{aligned} \quad (28)$$

where k_9 is arbitrary constant.

Assuming $M^2(\rho) = A(\rho)$ and $c = 0$; Eq. (28) transform into

$$\begin{aligned} (A')^2 = &-\frac{4h}{3a\mu^2}A^4 - \frac{2g}{a\mu^2}A^3 + \frac{4a\mu - \beta^2}{a^2\mu^4}A^2 \\ &+ k_9 A - \left(\frac{k_7^2}{a^2\beta^2\mu^4} - \frac{4b}{a\mu^2}\right). \end{aligned} \quad (29)$$

Equation (29) can be written as

$$(A')^2 = \alpha_1 A^4 + \alpha_2 A^3 + \alpha_3 A^2 + \alpha_4 A + \alpha_5, \quad (30)$$

where

$$\begin{aligned} &\alpha_1 = -\frac{4h}{3a\mu^2}, \quad \alpha_2 = -\frac{2g}{a\mu^2}, \quad \alpha_3 = \frac{4a\mu - \beta^2}{a^2\mu^4}, \\ &\alpha_4 = k_9, \quad \alpha_5 = -\left(\frac{k_7^2}{a^2\beta^2\mu^4} - \frac{4b}{a\mu^2}\right). \end{aligned} \quad (31)$$

(b) When $n = -1$, Eq. (26) can be written as

$$\begin{aligned} M'' = &\frac{1}{4}\frac{4a\mu - \beta^2}{a^2\mu^4}M + \frac{1}{4}\frac{k_7^2}{a^2\beta^2\mu^4}M^{-3} \\ &- \frac{1}{a\mu^2}\left(bM^{-1} + c + gM^2 + hM^3\right), \end{aligned} \quad (32)$$

Integrating (32), we have

$$\begin{aligned} \frac{(M')^2}{2} = &\frac{1}{8}\frac{4a\mu - \beta^2}{a^2\mu^4}M^2 - \frac{1}{8}\frac{k_7^2}{a^2\beta^2\mu^4}M^{-2} \\ &- \frac{1}{a\mu^2}\left(b\ln M + c + \frac{g}{3}M^3 + \frac{h}{4}M^4\right) + \frac{k_{10}}{2}, \end{aligned} \quad (33)$$

where k_{10} is arbitrary constant.

Assuming $M(\rho) = A(\rho)$, $k_7 = 0$ and $b = 0$; Eq. (33) transform into Eq. (30), where

$$\begin{aligned} &\alpha_1 = -\frac{h}{2a\mu^2}, \quad \alpha_2 = -\frac{2g}{3a\mu^2}, \quad \alpha_3 = \frac{4a\mu - \beta^2}{4a^2\mu^4}, \\ &\alpha_4 = -\frac{2c}{a\mu^2}, \quad \alpha_5 = k_{10}. \end{aligned} \quad (34)$$

(c) When $n = 1$, Eq. (26) can be written as

$$M'' = \frac{1}{4}\frac{4a\mu - \beta^2}{a^2\mu^4}M + \frac{1}{4}\frac{k_7^2}{a^2\beta^2\mu^4}M^{-3} - \frac{1}{a\mu^2}\left(bM^3 + cM^2 + g + hM^{-1}\right). \tag{35}$$

Integrating (35), we have

$$\frac{(M')^2}{2} = \frac{1}{8}\frac{4a\mu - \beta^2}{a^2\mu^4}M^2 - \frac{1}{8}\frac{k_7^2}{a^2\beta^2\mu^4}M^{-2} - \frac{1}{a\mu^2}\left(\frac{b}{4}\ln M + \frac{c}{3}M^3 + gM + hM^4\right) + \frac{k_{11}}{2}, \tag{36}$$

where k_{11} is arbitrary constant.

Assuming $M(\rho) = A(\rho)$, $k_7 = 0$ and $h = 0$; Eq. (36) transform into Eq. (30),where

$$\alpha_1 = -\frac{b}{2a\mu^2},\quad \alpha_2 = -\frac{2c}{3a\mu^2},\quad \alpha_3 = \frac{4a\mu - \beta^2}{4a^2\mu^4},\quad \alpha_4 = -\frac{2g}{a\mu^2},\quad \alpha_5 = k_{11}. \tag{37}$$

(d) When $n = 2$, Eq. (26) can be written as

$$M'' = \frac{1}{4}\frac{4a\mu - \beta^2}{a^2\mu^4}M + \frac{1}{4}\frac{k_7^2}{a^2\beta^2\mu^4}M^{-3} - \frac{1}{a\mu^2}\left(bM^5 + cM^3 + gM^{-1} + hM^{-3}\right). \tag{38}$$

Integrating (38), we have

$$\frac{(M')^2}{2} = \frac{1}{8}\frac{4a\mu - \beta^2}{a^2\mu^4}M^2 - \frac{1}{8}\frac{k_7^2}{a^2\beta^2\mu^4}M^{-2} - \frac{1}{a\mu^2}\left(\frac{b}{6}M^6 + \frac{c}{4}M^4 + g\ln M - \frac{h}{2}M^{-2}\right) + \frac{k_{12}}{8}, \tag{39}$$

where k_{12} is arbitrary constant.

Assuming $M^2(\rho) = A(\rho)$ and $g = 0$; Eq. (39) transform into Eq. (30), where

$$\alpha_1 = -\frac{4b}{3a\mu^2},\quad \alpha_2 = -\frac{2c}{a\mu^2},\quad \alpha_3 = \frac{4a\mu - \beta^2}{a^2\mu^4},\quad \alpha_4 = k_{12},\quad \alpha_5 = \frac{4h}{a\mu^2} - \frac{k_7^2}{a^2\beta^2\mu^4}. \tag{40}$$

(e) In the case of $n \neq -2, -1, 1, 2$, by integrating Eq. (26), we have

$$\frac{(M')^2}{2} = \frac{1}{8}\frac{4a\mu - \beta^2}{a^2\mu^4}M^2 - \frac{1}{a\mu^2}\left(\frac{b}{2n+2}M^{2n+2} + \frac{c}{n+2}M^{n+2} + \frac{g}{2-n}M^{2-n} + \frac{h}{2-2n}M^{2-2n}\right) - \frac{1}{8}\frac{k_7^2}{a^2\beta^2\mu^4}M^{-2} + \frac{k_{13}}{2}, \tag{41}$$

where k_{13} is arbitrary constant.

Let us assume $M(\rho) = A^{1/n}(\rho)$, we have

$$\frac{1}{2n^2}A^{2/n}(A')^2 = \frac{1}{8}\frac{4a\mu - \beta^2}{a^2\mu^4}A^{2/n} - \frac{1}{a\mu^2}\left(\frac{b}{2n+2}A^{2/n+2} + \frac{c}{n+2}A^{2/n+1} + \frac{g}{2-n}A^{2/n-1} + \frac{h}{2-2n}A^{2/n-2}\right) - \frac{1}{8}\frac{k_7^2}{a^2\beta^2\mu^4}A^{-2/n} + \frac{k_{13}}{2}. \tag{42}$$

Multiplying Eq. (42) by $n^2A^{2-2/n}$ and taking $k_7 = 0$ and $k_{13} = 0$, we have

$$(A')^2 = -\frac{bn^2}{a\mu^2(n+1)}A^4 - \frac{2cn^2}{a\mu^2(n+2)}A^3 + \frac{(4a\mu - \beta^2)n^2}{4a^2\mu^4}A^2 + \frac{2gn^2}{a\mu^2(n-2)}A + \frac{hn^2}{a\mu^2(n-1)}. \tag{43}$$

Equation (43) is similar to Eq. (30), where

$$\alpha_1 = -\frac{bn^2}{a\mu^2(n+1)},\quad \alpha_2 = -\frac{2cn^2}{a\mu^2(n+2)},\quad \alpha_3 = \frac{(4a\mu - \beta^2)n^2}{4a^2\mu^4},\quad \alpha_4 = \frac{2gn^2}{a\mu^2(n-2)},\quad \alpha_5 = \frac{hn^2}{a\mu^2(n-1)}. \tag{44}$$

Relation between parameters a, b, c, g, h, and n of (1) and parameters α_1, α_2, α_3, α_4, and α_5 of (30) in order to derive the solutions $M(\rho)$ of (26) is given by following Table 1.

3.3. Soliton Solutions to Equation (30)

To derive the general solution of the equation

$$(A')^2 = \alpha_1A^4 + \alpha_2A^3 + \alpha_3A^2 + \alpha_4A + \alpha_5, \tag{45}$$

one can use substituting methods [11, 12, 16, 17]. In [9], the author expressed the general solution of Eq. (45) in the form of Weierstrass and Jacobi elliptic function. Here we obtained the bright, dark and singular soliton solutions of Eq. (45).

3.3.1. Bright solitons. Let us assume

$$\alpha_4 = \frac{1}{8}\frac{\alpha_2(4\alpha_3\alpha_1 - \alpha_2^2)}{\alpha_1^2},\quad \alpha_5 = \frac{1}{256}\frac{\alpha_2^2(16\alpha_3\alpha_1 - 5\alpha_2^2)}{\alpha_1^3}.$$

Then the solution of Eq. (45) is given by

$$A(\rho) = -\frac{1}{4}\frac{\alpha_2}{\alpha_1} \pm \frac{1}{4\alpha_1}\sqrt{6\alpha_2^2 - 16\alpha_3\alpha_1} \times \operatorname{sech}\left[\frac{1}{4}\sqrt{\frac{2(8\alpha_3\alpha_1 - 3\alpha_2^2)}{\alpha_1}}\rho \pm k_{14}\right], \tag{46}$$

Table 1. Relation between parameters of Eqs. (1) and (30) in order to derive the solutions $M(\rho)$ of (26)

Parameters	α_1	α_2	α_3	α_4	α_5	$M(\rho)$
$n=-2$, $c=0$	$-\frac{4h}{3a\mu^2}$	$-\frac{2g}{a\mu^2}$	$\frac{4a\mu-\beta^2}{a^2\mu^4}$	k_9	$-\left(\frac{k_7^2}{a^2\beta^2\mu^4}-\frac{4b}{a\mu^2}\right)$	$\sqrt{A(\rho)}$
$n=-1$, $k_7=b=0$	$-\frac{h}{2a\mu^2}$	$-\frac{2g}{3a\mu^2}$	$\frac{4a\mu-\beta^2}{4a^2\mu^4}$	$-\frac{2c}{a\mu^2}$	k_{10}	$A(\rho)$
$n=1$, $k_7=h=0$	$-\frac{b}{2a\mu^2}$	$-\frac{2c}{3a\mu^2}$	$\frac{4a\mu-\beta^2}{4a^2\mu^4}$	$-\frac{2g}{a\mu^2}$	k_{11}	$A(\rho)$
$n=2$, $g=0$	$-\frac{4b}{3a\mu^2}$	$-\frac{2c}{a\mu^2}$	$\frac{4a\mu-\beta^2}{a^2\mu^4}$	k_{12}	$\frac{4h}{a\mu^2}-\frac{k_7^2}{a^2\beta^2\mu^4}$	$\sqrt{A(\rho)}$
$n\neq-2,-1,1,2$, $k_7=k_{13}=0$	$-\frac{bn^2}{a\mu^2(n+1)}$	$-\frac{2cn^2}{a\mu^2(n+2)}$	$\frac{(4a\mu-\beta^2)n^2}{4a^2\mu^4}$	$\frac{2gn^2}{a\mu^2(n-2)}$	$\frac{hn^2}{a\mu^2(n-1)}$	$A^{1/n}(\rho)$

where k_{14} is arbitrary constant. Hence corresponding bright soliton solution of Eq. (1) is given by

$$q(x,t) = M(\rho)\exp\left[i\left(\frac{x}{\beta}+\frac{\beta^2-2a\mu}{2\alpha\beta\mu^2}\int\frac{1}{M^2}d\rho+k_8\right)\right], \tag{47}$$

where $\rho=\mu x-\beta t$ and the values of M depending upon various parameters and the function A as discussed in Table 1 and A is given by (46).

3.3.2. Dark solitons. Let us assume

$$\alpha_4=\frac{1}{8}\frac{\alpha_2(4\alpha_3\alpha_1-\alpha_2^2)}{\alpha_1^2},\quad \alpha_5=\frac{1}{64}\frac{\alpha_2^2(4\alpha_3\alpha_1-\alpha_2^2)}{\alpha_1^3}.$$

Then the solution of Eq. (45) is given by

$$\begin{aligned}A(\rho) &= -\frac{1}{4}\frac{\alpha_2}{\alpha_1}\pm\frac{1}{4\alpha_1}\sqrt{3\alpha_2^2-8\alpha_3\alpha_1}\\ &\times\tanh\left[\frac{1}{4}\sqrt{\frac{3\alpha_2^2-8\alpha_3\alpha_1}{\alpha_1}}\rho\pm k_{15}\right],\end{aligned} \tag{48}$$

where k_{15} is arbitrary constant. Hence corresponding bright soliton solution of Eq. (1) is given by

$$\begin{aligned}q(x,t) &= M(\rho)\\ &\times\exp\left[i\left(\frac{x}{\beta}+\frac{\beta^2-2a\mu}{2\alpha\beta\mu^2}\rho+\frac{k_7}{2\alpha\beta\mu^2}\int\frac{1}{M^2}d\rho+k_8\right)\right],\end{aligned} \tag{49}$$

where $\rho=\mu x-\beta t$ and the values of M depending upon various parameters and the function A as discussed in Table 1 and A is given by (48).

3.3.3. Singular solitons. Let us assume

$$\alpha_4=\frac{1}{8}\frac{\alpha_2(4\alpha_3\alpha_1-\alpha_2^2)}{\alpha_1^2},\quad \alpha_5=\frac{1}{256}\frac{\alpha_2^2(16\alpha_3\alpha_1-5\alpha_2^2)}{\alpha_1^3}.$$

Then the solution of Eq. (45) is given by

$$\begin{aligned}A(\rho) &= -\frac{1}{4}\frac{\alpha_2}{\alpha_1}\pm\frac{1}{4\alpha_1}\sqrt{16\alpha_3\alpha_1-6\alpha_2^2}\\ &\times\operatorname{csch}\left[\frac{1}{4}\sqrt{\frac{2\left(8\alpha_3\alpha_1-3\alpha_2^2\right)}{\alpha_1}}\rho\pm k_{16}\right],\end{aligned} \tag{50}$$

where k_{16} is arbitrary constant. Hence corresponding bright soliton solution of Eq. (1) is given by

$$\begin{aligned}q(x,t) &= M(\rho)\\ &\times\exp\left[i\left(\frac{x}{\beta}+\frac{\beta^2-2a\mu}{2\alpha\beta\mu^2}\rho+\frac{k_7}{2\alpha\beta\mu^2}\int\frac{1}{M^2}d\rho+k_8\right)\right],\end{aligned} \tag{51}$$

where $\rho=\mu x-\beta t$ and the values of M depending upon various parameters and the function A as discussed in Table 1 and A is given by (50).

The constraint conditions or existence criteria for these solitons are given as

$$\alpha_1\left(8\alpha_3\alpha_1-3\alpha_2^2\right)>0, \tag{52}$$

for bright and singular solitons, while for dark solitons one needs to have

$$\alpha_1\left(8\alpha_3\alpha_1-3\alpha_2^2\right)<0. \tag{53}$$

4. CONCLUSIONS

This paper studied KE to secure bright, dark and singular soliton solutions to the model. The existence criteria for such solitons are also enumerated. These constraints guarantee the formation of such solitons. The results are thus strongly encouraging to pursue this avenue of research, further along. Later on, the model will be extended to study soliton dynamics with KE in birefringent fibers and DWDM topology. Additional optoelectronic devices are also going to be touched base upon. These would be magneto-optic waveguides, optical metamaterials, optical couplers and several such. Those results are currently awaited and they would be reported with time.

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CONFLICT OF INTEREST

The authors also declare that there is no conflict of interest.

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