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# Optical soliton polarization with Lakshmanan–Porsezian–Daniel model by unified approach

Mohammad Safi Ullah [a,b], Harun-Or-Roshid [c], M. Zulfikar Ali [b], Anjan Biswas [d,e,f,g], Mehmet Ekici [h,*], Salam Khan [d], Luminita Moraru [i], Abdullah Khamis Alzahrani [e], Milivoj R. Belic [j]

[a] Department of Mathematics, Comilla University, Cumilla 3506, Bangladesh
[b] Department of Mathematics, Rajshahi University, Rajshahi 6205, Bangladesh
[c] Department of Mathematics, Pabna University of Science and Technology, Pabna 6600, Bangladesh
[d] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762–4900, USA
[e] Mathematical Modeling and Applied Computation (MMAC) Research Group, Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[f] Department of Applied Mathematics, National Research Nuclear University, 31 Kashirskoe Hwy, Moscow 115409, Russian Federation
[g] Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Sciences University, Medunsa-0204, Pretoria, South Africa
[h] Department of Mathematics, Faculty of Science and Arts, Yozgat Bozok University, 66100 Yozgat, Turkey
[i] Faculty of Sciences and Environment, Department of Chemistry, Physics and Environment, Dunarea de Jos University of Galati, 47 Domneasca Street, 800008, Romania
[j] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

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ABSTRACT

This work retrieves polarized optical soliton solutions for pulses in birefringent fibers that are modeled by the Lakshmanan–Porsezian–Daniel model. The unified approach recovers singular solitons only. This approach fails to retrieve the much needed bright and dark soliton solutions. These singular solitons exist with restricted parametric conditions that are also exhibited.

## Introduction

Optical soliton dynamics is an engineering marvel in telecommunications industry [1–10]. An inherent problem with the dynamics of pulse propagation across trans-oceanic and trans-continental distances is its polarization. This is attributed to several factors such as the randomness of fiber diameter, rough handling of optical fibers and many others. These factors occasionally lead to hi-bi fibers. It is often a challenging task to retrieve the soliton solutions to the models that are studied in the context of high birefringence.

One such model that has been around for a fairly long period is the Lakshmanan–Porsezian–Daniel (LPD) model that was first reported in 1988 and later gained a lot of popularity [11]. A wide range of integration algorithms have been implemented to secure soliton and other solutions to LPD model in the context of polarization-preserving fibers [12], including $\exp(-\phi(\xi))$-expansion scheme [13], trial equation scheme [14] and many more [15–18]. Today's work will retrieve soliton solutions to LPD model with differential group delay by unified approach that was first reported during 2018 [19]. As it will be revealed, the algorithm could only expose singular solitons. The details are jotted in the rest of the paper after a quick re-visitation of the model and the integration algorithm.

### Governing model

The dimensionless LPD model with Kerr law nonlinearity has the following form [15,16,20]:

$$i\psi_t + a\psi_{xx} + b\psi_{xt} + c|\psi|^2\psi = \sigma\psi_{xxxx} + p\psi_x^2\psi^* + q|\psi_x|^2\psi + r|\psi|^2\psi_{xx} + \lambda\psi^2\psi_{xx}^* + s|\psi|^4\psi. \tag{1}$$

In Eq. (1), $x$ and $t$ represent independent spatial and temporal variables, respectively. The dependent variable $\psi$ represents the complex wave

* Corresponding author.
*E-mail address:* mehmet.ekici@bozok.edu.tr (M. Ekici).

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function. Next, the parameters $a$, $b$, $c$, $\sigma$ and $s$ signify group velocity dispersion, spatio-temporal dispersion, the coefficient of Kerr law nonlinearity, the coefficient of fourth order dispersion and the two-photon absorption, respectively. Finally, the $p$, $q$, $r$ and $\lambda$ terms account for several forms of the nonlinear dispersion. Solitons are possible for a sustained delicate balance of dispersion with the nonlinear terms.

For birefringent fibers, the model can be divided into two parts of a vector representation. Avoiding the properties of 4WM, the above model reduces to [13,14]:

$$\begin{aligned} &iu_t + a_1u_{xx} + b_1u_{xt} + \left(c_1 |u|^2 + d_1 |v|^2\right) u \\ &= \sigma_1 u_{xxxx} + \left(p_1 u_x^2 + q_1 v_x^2\right) u^* + \left(r_1 |u_x|^2 + s_1 |v_x|^2\right) u \\ &+ \left(\lambda_1 |u|^2 + \theta_1 |v|^2\right) u_{xx} + \left(\chi_1 u^2 + \eta_1 v^2\right) u^*_{xx} \\ &+ \left(f_1 |u|^4 + \phi_1 |u|^2 |v|^2 + \vartheta_1 |v|^4\right) u \end{aligned} \tag{2}$$

$$\begin{aligned} &iv_t + a_2v_{xx} + b_2v_{xt} + \left(c_2 |v|^2 + d_2 |u|^2\right) v \\ &= \sigma_2 v_{xxxx} + \left(p_2 v_x^2 + q_2 u_x^2\right) v^* + \left(r_2 |v_x|^2 + s_2 |u_x|^2\right) v \\ &+ \left(\lambda_2 |v|^2 + \theta_2 |u|^2\right) v_{xx} + \left(\chi_2 v^2 + \eta_2 u^2\right) v^*_{xx} \\ &+ \left(f_2 |v|^4 + \phi_2 |v|^2 |u|^2 + \vartheta_2 |u|^4\right) v. \end{aligned} \tag{3}$$

In Eqs. (2) and (3), $c_j, f_j$ for $j = 1,2$ represent the self-phase and $d_j, \phi_j, \vartheta_j$ with $j = 1,2$ stand for the cross-phase modulation effects, respectively.

### Mathematical analysis

Consider the following transformation of this coupled system

$$u(x,t) = H_1(\zeta)\exp(i\varphi) \tag{4}$$

$$v(x,t) = H_2(\zeta)\exp(i\varphi) \tag{5}$$

where $H_1$ and $H_2$ are the soliton amplitude components and

$$\zeta = x - \varpi t \tag{6}$$

is the traveling wave variable with the soliton speed $\varpi$. The phase component $\varphi$ is as below:

$$\varphi = -kx + wt + \varepsilon \tag{7}$$

with frequency $k$, wave number $w$ and phase shift $\varepsilon$. Inserting Eqs. (4) and (5) into Eqs. (2) and (3) and sorting out the real and imaginary parts leads to the following equations. The real part is

$$\begin{aligned} &(w + a_nk^2 - b_nkw + k^4\sigma_n)H_n - (c_n + k^2(p_n - r_n + \lambda_n + \chi_n))H_n^3 \\ &+ f_nH_n^5 - (d_n + k^2(q_n - s_n + \eta_n + \theta_n))H_n(H_{\bar{n}})^2 + \phi_n(H_n)^3(H_{\bar{n}})^2 \\ &+ \vartheta_nH_n(H_{\bar{n}})^4 + (p_n + r_n)H_n(H_n')^2 + (q_n + s_n)H_n(H_{\bar{n}}')^2 \\ &- (a_n - b_n\varpi + 6k^2\sigma_n)H_n'' + (\lambda_n + \chi_n)H_n^2H_n'' \\ &+ (\eta_n + \theta_n)(H_{\bar{n}})^2H_n'' + \sigma_nH_n^{(4)} = 0 \end{aligned} \tag{8}$$

while the imaginary part is

$$\begin{aligned} &(\varpi + 2a_nk - b_n(k\varpi + w) + 4k^3\sigma_n)H_n' - 2k(p_n + \lambda_n - \chi_n)H_n^2H_n' \\ &+ 2k(\eta_n - \theta_n)H_n'(H_{\bar{n}})^2 - 2q_nkH_nH_{\bar{n}}H_{\bar{n}}' - 4k\sigma_nH_n^{(3)} = 0 \end{aligned} \tag{9}$$

with $n = 1,2$ and $\bar{n} = 3 - n$. By the balancing principle, one can write

$$H_{\bar{n}} = H_n. \tag{10}$$

From Eqs. (8) and (10), we can rewrite

$$\begin{aligned} &(w + a_nk^2 - b_nkw + k^4\sigma_n)H_n - (c_n + d_n + k^2(h_n + R_n))H_n^3 + J_nH_n^5 \\ &+ L_nH_n(H_n')^2 - (a_n - b_n\varpi + 6k^2\sigma_n)H_n'' + R_nH_n^2H_n'' + \sigma_nH_n^{(4)} = 0 \end{aligned} \tag{11}$$

where

$$J_n = f_n + \phi_n + \vartheta_n, \quad h_n = p_n + q_n - r_n - s_n,$$

$$L_n = p_n + q_n + r_n + s_n, \quad R_n = \eta_n + \theta_n + \lambda_n + \chi_n. \tag{12}$$

From Eqs. (9) and (10), one can rewrite

$$\begin{aligned} &(\varpi + 2a_nk - b_n(k\varpi + w) + 4k^3\sigma_n)H_n' \\ &- 2k(p_n + q_n - \eta_n + \theta_n + \lambda_n - \chi_n)H_n^2H_n' - 4k\sigma_nH_n^{(3)} = 0. \end{aligned} \tag{13}$$

Thus, the third expression of Eq. (13) gives $\sigma_n = 0$. Hence the solutions of the coupled system Eqs. (2) and (3) will be presented for the fourth order dispersion omitted. The other terms in Eq. (13), yield the following relation

$$\eta_n + \chi_n = p_n + q_n + \theta_n + \lambda_n \tag{14}$$

and therefore the soliton speed is

$$\varpi = \frac{v_nw - 2a_nk}{1 - v_nk} \tag{15}$$

for $b_n = \frac{1}{k}$. Comparing the values of the soliton velocity, Eq. (15) gives

$$(1 - b_1k)(b_2w - 2a_2k) = (1 - b_2k)(b_1w - 2a_1k). \tag{16}$$

Therefore Eq. (11) can be written as

$$\begin{aligned} &(w + a_nk^2 - b_nkw)H_n - (c_n + d_n + k^2(h_n + R_n))H_n^3 + J_nH_n^5 + L_nH_n(H_n')^2 \\ &- (a_n - b_n\varpi)H_n'' + R_nH_n^2H_n'' = 0. \end{aligned} \tag{17}$$

### Application of unified method to LPD model

Assume the trial solution of Eq. (17) is

$$H_n(\zeta) = \sum_{i=0}^{N} A_i^{(n)}S(\zeta)^i + B_i^{(n)}S(\zeta)^{-i} \tag{18}$$

where $A_0^{(n)}, A_i^{(n)}$ and $B_i^{(n)}$ for $i = 1,2,\ldots,N$ are real constants and $S(\zeta)$ satisfies Riccati equation:

$$S'(\zeta) = S^2(\zeta) + l. \tag{19}$$

Eq. (19) has nine solution categories according to three cases:

**Case-1:** Hyperbolic functions (when $l < 0$):

$$S(\zeta) = \begin{cases} \dfrac{\sqrt{-(C^2 + D^2)l} - C\sqrt{-l}\cosh(2\sqrt{-l}(\zeta + E))}{C\sinh(2\sqrt{-l}(\zeta + E)) + D}, \\ \dfrac{-\sqrt{-(C^2 + D^2)l} - C\sqrt{-l}\cosh(2\sqrt{-l}(\zeta + E))}{C\sinh(2\sqrt{-l}(\zeta + E)) + D}, \\ \sqrt{-l} + \dfrac{2C\sqrt{-l}}{C + \cosh(2\sqrt{-l}(\zeta + E)) - \sinh(2\sqrt{-l}(\zeta + E))}, \\ -\sqrt{-l} + \dfrac{2C\sqrt{-l}}{C + \cosh(2\sqrt{-l}(\zeta + E)) - \sinh(2\sqrt{-l}(\zeta + E))}, \end{cases} \tag{20}$$

**Case-2:** Trigonometric functions (when $l > 0$):

$$S(\zeta) = \begin{cases} \dfrac{\sqrt{(C^2 - D^2)l} - C\sqrt{l}\cos(2\sqrt{l}(\zeta + E))}{C\sin(2\sqrt{l}(\zeta + E)) + D}, \\ \dfrac{-\sqrt{(C^2 - D^2)l} - C\sqrt{l}\cos(2\sqrt{l}(\zeta + E))}{C\sin(2\sqrt{l}(\zeta + E)) + D}, \\ i\sqrt{l} + \dfrac{-2iC\sqrt{l}}{C + cos(2\sqrt{l}(\zeta + E)) - i\sin(2\sqrt{l}(\zeta + E))}, \\ -i\sqrt{l} + \dfrac{2iC\sqrt{l}}{C + cos(2\sqrt{l}(\zeta + E)) - i\sin(2\sqrt{l}(\zeta + E))}, \end{cases} \tag{21}$$

where $C \neq 0$ and $D$, $E$ are real arbitrary constants.

**Case-3:** Rational function solution (when $l = 0$):

$$S(\zeta) = \frac{1}{\zeta + E}. \tag{22}$$

To identify the value of $N$ in Eq. (18), balancing $H_n^2 H_n''$ with $H_n^5$ yields $N = 1$. Eq. (18) takes the form

$$H_n(\zeta) = A_0^{(n)} + A_1^{(n)} S(\zeta) + B_1^{(n)} S(\zeta)^{-1}. \tag{23}$$

Then putting Eq. (23) along with Eq. (19) into Eq. (17) and after some calculations, we pose the following sets of solutions:

**Set-1:** $A_0^{(n)} = 0,\ A_1^{(n)} = M_n,\ B_1^{(n)} = 0$

$$\begin{aligned} w =\ & (k^3 J_n a_n b_n - k\lambda^2 L_n^2 b_n - 2kl^2 L_n R_n b_n + 2klJ_n a_n b_n - k^2 J_n a_n \\ & + l^2 L_n^2 + 2l^2 L_n R_n + 2lJ_n a_n)/(J_n(k^2 b_n^2 + 2lb_n^2 - 2kb_n + 1)) \end{aligned}$$

$$\begin{aligned} c_n =\ & -(k^4 h_n L_n b_n^2 + 2k^4 h_n R_n b_n^2 + k^4 L_n R_n b_n^2 + 2k^4 R_n^2 b_n^2 + 2k^2 l h_n L_n b_n^2 \\ & + 4k^2 l h_n R_n b_n^2 - 2k^2 l L_n^2 b_n^2 - 4k^2 l L_n R_n b_n^2 - 2k^3 h_n L_n b_n - 4k^3 h_n R_n b_n \\ & - 2k^3 L_n R_n b_n - 4k^3 R_n^2 b_n + k^2 L_n b_n^2 d_n + 2k^2 R_n b_n^2 d_n - 2l^2 L_n^2 b_n^2 \\ & - 8l^2 L_n R_n b_n^2 - 8l^2 R_n^2 b_n^2 + 4klL_n^2 b_n + 12klL_n R_n b_n + 8klR_n^2 b_n + 2lL_n b_n^2 d_n \\ & + 4lR_n b_n^2 d_n + k^2 h_n L_n + 2k^2 h_n R_n + k^2 L_n R_n + 2k^2 R_n^2 - 2kL_n b_n d_n \\ & - 4kR_n b_n d_n - 2lL_n^2 - 6lL_n R_n - 4lR_n^2 - 2J_n a_n + L_n d_n + 2R_n d_n)/(k^2 L_n b_n^2 \\ & + 2k^2 R_n b_n^2 + 2lL_n b_n^2 + 4lR_n b_n^2 - 2kL_n b_n - 4kR_n b_n + L_n + 2R_n) \end{aligned}$$

**Set-2:** $A_0^{(n)} = 0,\ A_1^{(n)} = 0,\ B_1^{(n)} = M_n l$

$$\begin{aligned} w =\ & (k^3 J_n a_n b_n - k\lambda^2 L_n^2 b_n - 2kl^2 L_n R_n b_n + 2klJ_n a_n b_n - k^2 J_n a_n + l^2 L_n^2 \\ & + 2l^2 L_n R_n + 2lJ_n a_n)/(J_n(k^2 b_n^2 + 2lb_n^2 - 2kb_n + 1)) \end{aligned}$$

$$\begin{aligned} c_n =\ & -(k^4 h_n L_n b_n^2 + 2k^4 h_n R_n b_n^2 + k^4 L_n R_n b_n^2 + 2k^4 R_n^2 b_n^2 + 2k^2 l h_n L_n b_n^2 \\ & + 4k^2 l h_n R_n b_n^2 - 2k^2 l L_n^2 b_n^2 - 4k^2 l L_n R_n b_n^2 - 2k^3 h_n L_n b_n - 4k^3 h_n R_n b_n \\ & - 2k^3 L_n R_n b_n - 4k^3 R_n^2 b_n + k^2 L_n b_n^2 d_n + 2k^2 R_n b_n^2 d_n - 2l^2 L_n^2 b_n^2 \\ & - 8l^2 L_n R_n b_n^2 - 8l^2 R_n^2 b_n^2 + 4klL_n^2 b_n + 12klL_n R_n b_n + 8klR_n^2 b_n + 2lL_n b_n^2 d_n \\ & + 4lR_n b_n^2 d_n + k^2 h_n L_n + 2k^2 h_n R_n + k^2 L_n R_n + 2k^2 R_n^2 - 2kL_n b_n d_n \\ & - 4kR_n b_n d_n - 2lL_n^2 - 6lL_n R_n - 4lR_n^2 - 2J_n a_n + L_n d_n + 2R_n d_n)/(k^2 L_n b_n^2 \\ & + 2k^2 R_n b_n^2 + 2lL_n b_n^2 + 4lR_n b_n^2 - 2kL_n b_n - 4kR_n b_n + L_n + 2R_n) \end{aligned}$$

**Set-3:** $A_0^{(n)} = 0,\ A_1^{(n)} = M_n,\ B_1^{(n)} = M_n l$

$$\begin{aligned} w =\ & -(8M_n^2 l^2 kJ_n L_n^2 b_n + 16M_n^2 l^2 kJ_n L_n R_n b_n - 6M_n^2 lkJ_n^2 a_n b_n - k^3 J_n L_n a_n b_n \\ & - 2k^3 J_n R_n a_n b_n + 8kl^2 L_n^3 b_n + 32l^2 L_n^2 R_n b_n + 32kl^2 L_n R_n^2 b_n - 8M_n^2 l^2 J_n L_n^2 \\ & - 16M_n^2 l^2 J_n L^n R_n - 2klJ_n L_n a_n b_n - 4klJ_n R_n a_n b_n - 6M_n^2 lJ_n^2 a_n + k^2 J_n a_n \\ & + 2k^2 J_n R_n a_n - 8l^2 L_n^3 - 32l^2 L_n^2 R_n - 32l^2 L_n R_n^2 - 2lJ_n L_n a_n - 4lJ_n R_n a_n) \\ & /(J_n(6J_n b_n^2 M_n^2 l + k^2 L_n b_n^2 + 2k^2 R_n b_n^2 + 2lL_n b_n^2 + 4lR_n b_n^2 - 2kL_n b_n \\ & - 4kR_n b_n + L_n + 2R_n)) \end{aligned}$$

$$\begin{aligned} c_n =\ & -(6k^2 h_n J_n b_n^2 M_n^2 - 6k^2 J_n L_n b_n^2 M_n^2 l + 12kJ_n L_n b_n M_n^2 l + 12kJ_n R_n b_n M_n^2 l \\ & + 2k^2 l h_n L_n b_n^2 + 4k^2 l h_n R_n b_n^2 - 4k^2 l L_n R_n b_n^2 + 12klL_n R_n b_n - 6J_n L_n M_n^2 l \\ & - 6J_n R_n M_n^2 l - 2a_n J_n + 2k^2 R_n^2 - 2lL_n^2 - 4lR_n^2 + L_n d_n + 2R_n d_n - 24l^2 L_n^2 b_n^2 \\ & + 2k^4 R_n^2 b_n^2 - 4k^3 R_n^2 b_n - 80l^2 R_n^2 b_n^2 + k^2 h_n L_n + 2k^2 h_n r_n + k^2 L_n R_n - 6lL_n R_n \\ & - 88l^2 L_n R_n b_n^2 + k^4 h_n L_n b_n^2 + 2k^4 h_n R_n b_n^2 + k^4 L_n R_n b_n^2 - 2k^2 l L_n^2 b_n^2 - 2k^3 h_n L_n b_n \\ & - 4k^3 h_n R_n b_n - 2k^3 L_n R_n b_n + k^2 L_n b_n^2 d_n + 2k^2 R_n b_n^2 d_n + 4klL_n^2 b_n + 8klR_n^2 b_n \\ & + 2lL_n b_n^2 d_n + 4lR_n b_n^2 d_n - 2kL_n b_n d_n - 4kR_n b_n d_n - 8l^2 J_n b_n^2 M^2 - 24l^2 J_n R_n b_n^2 M_n^2 \\ & + 6J_n b_n^2 d_n M_n^2 l)/(6J_n b_n^2 M_n^2 l + k^2 L_n b_n^2 + 2k^2 R_n b_n^2 + 2lL_n b_n^2 + 4lR_n b_n^2 - 2kL_n b_n \\ & - 4kR_n b_n + L_n + 2R_n) \end{aligned}$$

for

$$M_n = \pm \frac{\sqrt{-J_n(L_n + 2R_n)}}{J_n}.$$

Using Eqs. (20)–(21)–(22) and Eqs. (4)–(5), with the help of the solution Set-1, we obtain the following eighteen exact solutions of Eqs. (2) and (3):

$$u_{1,1}(x,t) = M_1 \left( \frac{\sqrt{-(C^2 + D^2)l} - C\sqrt{-l}\cosh(2\sqrt{-l}(\zeta + E))}{C\sinh(2\sqrt{-l}(\zeta + E)) + D} \right) \exp[i\varphi]$$

$$v_{1,1}(x,t) = M_2 \left( \frac{\sqrt{-(C^2 + D^2)l} - C\sqrt{-l}\cosh(2\sqrt{-l}(\zeta + E))}{C\sinh(2\sqrt{-l}(\zeta + E)) + D} \right) \exp[i\varphi]$$

$$u_{1,2}(x,t) = M_1 \left( \frac{-\sqrt{-(C^2 + D^2)l} - C\sqrt{-l}\cosh(2\sqrt{-l}(\zeta + E))}{C\sinh(2\sqrt{-l}(\zeta + E)) + D} \right) \exp[i\varphi]$$

$$v_{1,2}(x,t) = M_2 \left( \frac{-\sqrt{-(C^2 + D^2)l} - C\sqrt{-l}\cosh(2\sqrt{-l}(\zeta + E))}{C\sinh(2\sqrt{-l}(\zeta + E)) + D} \right) \exp[i\varphi]$$

$$u_{1,3}(x,t) = M_1 \left( \sqrt{-l} + \frac{2C\sqrt{-l}}{C + \cosh(2\sqrt{-l}(\zeta + E)) - \sinh(2\sqrt{-l}(\zeta + E))} \right) \exp[i\varphi]$$

$$v_{1,3}(x,t) = M_2 \left( \sqrt{-l} + \frac{2C\sqrt{-l}}{C + \cosh(2\sqrt{-l}(\zeta + E)) - \sinh(2\sqrt{-l}(\zeta + E))} \right) \exp[i\varphi]$$

$$u_{1,4}(x,t) = M_1 \left( -\sqrt{-l} + \frac{2C\sqrt{-l}}{C + \cosh(2\sqrt{-l}(\zeta + E)) - \sinh(2\sqrt{-l}(\zeta + E))} \right) \exp[i\varphi]$$

$$v_{1,4}(x,t) = M_2 \left( -\sqrt{-l} + \frac{2C\sqrt{-l}}{C + \cosh(2\sqrt{-l}(\zeta + E)) - \sinh(2\sqrt{-l}(\zeta + E))} \right) \exp[i\varphi]$$

which are all singular soliton pairs. Further, one arrives at the following periodic solutions:

$$u_{1,5}(x,t) = M_1 \left( \frac{\sqrt{(C^2 - D^2)l} - C\sqrt{l}\cos(2\sqrt{l}(\zeta + E))}{C\sin(2\sqrt{l}(\zeta + E)) + D} \right) \exp[i\varphi]$$

$$v_{1,5}(x,t) = M_2 \left( \frac{\sqrt{(C^2 - D^2)l} - C\sqrt{l}\cos(2\sqrt{l}(\zeta + E))}{C\sin(2\sqrt{l}(\zeta + E)) + D} \right) \exp[i\varphi]$$

$$u_{1,6}(x,t) = M_1 \left( \frac{-\sqrt{(C^2 - D^2)l} - C\sqrt{l}\cos(2\sqrt{l}(\zeta + E))}{C\sin(2\sqrt{l}(\zeta + E)) + D} \right) \exp[i\varphi]$$

$$v_{1,6}(x,t) = M_2 \left( \frac{-\sqrt{(C^2 - D^2)l} - C\sqrt{l}\cos(2\sqrt{l}(\zeta + E))}{C\sin(2\sqrt{l}(\zeta + E)) + D} \right) \exp[i\varphi]$$

$$u_{1,7}(x,t) = M_1 \left( i\sqrt{l} + \frac{-2iC\sqrt{l}}{C + \cos(2\sqrt{l}(\zeta + E)) - i\sin(2\sqrt{l}(\zeta + E))} \right) \exp[i\varphi]$$

$$v_{1,7}(x,t) = M_2 \left( i\sqrt{l} + \frac{-2iC\sqrt{l}}{C + \cos(2\sqrt{l}(\zeta + E)) - i\sin(2\sqrt{l}(\zeta + E))} \right) \exp[i\varphi]$$

$$u_{1,8}(x,t) = M_1 \left( -i\sqrt{l} + \frac{2iC\sqrt{l}}{C + \cos(2\sqrt{l}(\zeta + E)) - i\sin(2\sqrt{l}(\zeta + E))} \right) \exp[i\varphi]$$

$$v_{1,8}(x,t) = M_2 \left( -i\sqrt{l} + \frac{2iC\sqrt{l}}{C + \cos(2\sqrt{l}(\zeta + E)) - i\sin(2\sqrt{l}(\zeta + E))} \right) \exp[i\varphi]$$

$$u_{1,9}(x,t) = \frac{M_1}{\zeta + E}\exp[i\varphi]$$

$$v_{1,9}(x,t) = \frac{M_2}{\zeta + E}\exp[i\varphi]$$

where $M_n$ and $w$ are come from Set-1 and $\varphi = -kx + wt + \varepsilon$. Here $u_{1,9}$ and $v_{1,9}$ are the rational solutions to the model.

Using Eqs. (20)–(21)–(22) and Eqs. (4)–(5), with the help of the solution Set-2, we obtain the following eighteen exact solutions of Eqs. (2) and (3):

$$u_{2,1}(x,t) = \frac{M_1 l}{\dfrac{\sqrt{-(C^2 + D^2)l} - C\sqrt{-l}\cosh(2\sqrt{-l}(\zeta + E))}{C\sinh(2\sqrt{-l}(\zeta + E)) + D}} \exp[i\varphi]$$

$$v_{2,1}(x,t) = \frac{M_2 l}{\dfrac{\sqrt{-(C^2 + D^2)l} - C\sqrt{-l}\cosh(2\sqrt{-l}(\zeta + E))}{C\sinh(2\sqrt{-l}(\zeta + E)) + D}} \exp[i\varphi]$$

$$u_{2,2}(x,t) = \frac{M_1 l}{\dfrac{-\sqrt{-(C^2 + D^2)l} - C\sqrt{-l}\cosh(2\sqrt{-l}(\zeta + E))}{C\sinh(2\sqrt{-l}(\zeta + E)) + D}} \exp[i\varphi]$$

$$v_{2,2}(x,t) = \frac{M_2 l}{\dfrac{-\sqrt{-(C^2 + D^2)l} - C\sqrt{-l}\cosh(2\sqrt{-l}(\zeta + E))}{C\sinh(2\sqrt{-l}(\zeta + E)) + D}} \exp[i\varphi]$$

$$u_{2,3}(x,t)=\frac{M_1 l}{\sqrt{-l}+\dfrac{2C\sqrt{-l}}{C+\cosh(2\sqrt{-l}(\zeta+E))-\sinh(2\sqrt{-l}(\zeta+E))}}\exp[i\varphi]$$
$$v_{2,3}(x,t)=\frac{M_2 l}{\sqrt{-l}+\dfrac{2C\sqrt{-l}}{C+\cosh(2\sqrt{-l}(\zeta+E))-\sinh(2\sqrt{-l}(\zeta+E))}}\exp[i\varphi]$$
$$u_{2,4}(x,t)=\frac{M_1 l}{-\sqrt{-l}+\dfrac{2C\sqrt{-l}}{C+\cosh(2\sqrt{-l}(\zeta+E))-\sinh(2\sqrt{-l}(\zeta+E))}}\exp[i\varphi]$$
$$v_{2,4}(x,t)=\frac{M_2 l}{-\sqrt{-l}+\dfrac{2C\sqrt{-l}}{C+\cosh(2\sqrt{-l}(\zeta+E))-\sinh(2\sqrt{-l}(\zeta+E))}}\exp[i\varphi]$$

These constitute another set of singular solitons. Next, the periodic solution pairs are

$$u_{2,5}(x,t)=\frac{M_1 l}{\dfrac{\sqrt{(C^2-D^2)l}-C\sqrt{l}\cos(2\sqrt{l}(\zeta+E))}{C\sin(2\sqrt{l}(\zeta+E))+D}}\exp[i\varphi]$$
$$v_{2,5}(x,t)=\frac{M_2 l}{\dfrac{\sqrt{(C^2-D^2)l}-C\sqrt{l}\cos(2\sqrt{l}(\zeta+E))}{C\sin(2\sqrt{l}(\zeta+E))+D}}\exp[i\varphi]$$
$$u_{2,6}(x,t)=\frac{M_1 l}{\dfrac{-\sqrt{(C^2-D^2)l}-C\sqrt{l}\cos(2\sqrt{l}(\zeta+E))}{C\sin(2\sqrt{l}(\zeta+E))+D}}\exp[i\varphi]$$
$$v_{2,6}(x,t)=\frac{M_2 l}{\dfrac{-\sqrt{(C^2-D^2)l}-C\sqrt{l}\cos(2\sqrt{l}(\zeta+E))}{C\sin(2\sqrt{l}(\zeta+E))+D}}\exp[i\varphi]$$

$$u_{2,7}(x,t)=\frac{M_1 l}{i\sqrt{l}+\dfrac{-2iC\sqrt{l}}{C+\cos(2\sqrt{l}(\zeta+E))-i\sin(2\sqrt{l}(\zeta+E))}}\exp[i\varphi]$$
$$v_{2,7}(x,t)=\frac{M_2 l}{i\sqrt{l}+\dfrac{-2iC\sqrt{l}}{C+\cos(2\sqrt{l}(\zeta+E))-i\sin(2\sqrt{l}(\zeta+E))}}\exp[i\varphi]$$
$$u_{2,8}(x,t)=\frac{M_1 l}{-i\sqrt{l}+\dfrac{2iC\sqrt{l}}{C+\cos(2\sqrt{l}(\zeta+E))-i\sin(2\sqrt{l}(\zeta+E))}}\exp[i\varphi]$$
$$v_{2,8}(x,t)=\frac{M_2 l}{-i\sqrt{l}+\dfrac{2iC\sqrt{l}}{C+\cos(2\sqrt{l}(\zeta+E))-i\sin(2\sqrt{l}(\zeta+E))}}\exp[i\varphi]$$
$$u_{2,9}(x,t)=M_1 l(\zeta+E)\exp[i\varphi]$$
$$v_{2,9}(x,t)=M_2 l(\zeta+E)\exp[i\varphi]$$

where $M_n$ and $w$ are come from Set-2 and $\varphi=-kx+wt+\varepsilon$.

Similarly, using Eqs. (20)–(21) and Eqs. (4)–(5), with the help of the solution Set-3, we obtain the sixteen exact solutions of Eqs. (2) and (3). But, Eq. (22) does not provide any exact solution of Eqs. (2) and (3) for the solution Set-3.

## Conclusions

This paper revealed soliton solutions to LPD model with differential group delay. The polarized solitons are thus retrieved and exhibited. The scheme implemented is the unified approach which yielded singular soliton solutions only. Singular solitons are applicable to model optical rogons, but not optical solitons, and the algorithm, evidently, has a few drawbacks. The method fails to retrieve the much-needed bright solitons and dark solitons. Also, this scheme is unable to produce $N$-soliton solutions to the governing model. Moreover, a profound drawback is its inability to locate soliton radiation that is inevitably present once linear and nonlinear dispersion terms are embedded in the model. Thus, to conclude, the unified approach is not of much use in the study of governing models that give rise to optical solitons. From the applications perspective, this integration algorithm cannot be applied to obtain bright or dark solitons in any model. It can only be used to address optical rogons that are supposedly modeled with singular solitons. In addition, singular solitons cannot be plotted. This paper therefore concludes with analytical results for only singular solitons obtained by the aid of the unified method.

## CRediT authorship contribution statement

**Mohammad Safi Ullah:** Methodology, Software. **Harun-Or-Roshid:** Methodology, Software. **M. Zulfikar Ali:** Methodology, Software. **Anjan Biswas:** Supervision. **Mehmet Ekici:** Investigation. **Salam Khan:** Supervision. **Luminita Moraru:** Validation. **Abdullah Khamis Alzahrani:** Validation. **Milivoj R. Belic:** Funding acquisition.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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