Journal of Nonlinear Optical Physics & Materials
Vol. 30, Nos. 3&4 (2021) 2150005 (17 pages)

DOI: 10.1142/S0218863521500053

World Scientific
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Cubic–quartic polarized optical solitons and conservation laws for perturbed Fokas–Lenells model

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Received 6 February 2021
Revised 30 March 2021
Accepted 16 July 2021
Published 8 September 2021

This paper studies polarized cubic–quartic solitons that are modeled by Fokas–Lenells equation in birefringent fibers. Two integration schemes recovered a spectrum of soliton solutions to the model. Subsequently, the bright solitons compute the corresponding conserved quantities from the respective densities that are recovered by the multiplier approach.

Keywords: Solitons; polarization; Fokas–Lenells.

OCIS Codes: 060.2310, 060.4510, 060.5530, 190.3270, 190.4370.

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1. Introduction

The concept of cubic–quartic (CQ) optical solitons first appeared a couple of years ago as a generalization to pure-quartic solitons and pure-cubic solitons that were introduced to sustain the delicate balance between chromatic dispersion (CD) and nonlinearity. The issue with pure-quartic solitons is that an analytical soliton solution cannot be established unless one is looking for a stationary soliton solution. On the other hand, pure-cubic soliton model is not integrable unless Hamiltonian perturbation terms are included in it. This led to the novel concept of CQ solitons that has achieved success in the fields of quantum optics and telecommunication industry. In CQ solitons, CD is replaced by a combination of third-order dispersion (3OD) and fourth-order dispersion (4OD) that can together replenish the low count of CD. A plethora of results are reported during the past decade or so in these arenas.[1–27]

One of the familiar models that addresses the dynamics of soliton propagation through optical fibers and PCF, apart from the familiar nonlinear Schrödinger's equation, is Fokas–Lenells equation (FLE). This model is well studied in the context of polarization-preserving fibers with CD and/or CQ replacing CD. Today's work turns the page to extend the study of CQ solitons with FLE for polarized pulses. Thus, the model equations for CQ solitons in birefringent fibers are first introduced. Subsequently, a couple of familiar integration algorithms would give way to a spectrum of CQ soliton solutions to FLE in birefringent fibers. Finally, the paper enumerates and exhibits conservation laws to the model that are recovered by the aid of multiplier approach. The corresponding conserved quantities are eventually computed and presented. These results are displayed after a succinct intro to the governing model.

1.1. *Governing model*

The CQ in birefringent fibers for the perturbed FLE reads as follows:

$$\begin{aligned} &iq_t + ia_1 q_{xxx} + b_1 q_{xxxx} + (c_1|q|^2 + d_1|r|^2)(e_1 q + if_1 q_x) + qr^*(\gamma_1 r + i\eta_1 r_x) \\ &\quad = i[\delta_1 q_x + \lambda_1(|q|^2 q)_x + \mu_1(|q|^2)_x q], \end{aligned} \tag{1}$$

$$\begin{aligned} &ir_t + ia_2 r_{xxx} + b_2 r_{xxxx} + (c_2|r|^2 + d_2|q|^2)(e_2 r + if_2 r_x) + rq^*(\gamma_2 q + i\eta_2 q_x) \\ &\quad = i[\delta_2 r_x + \lambda_2(|r|^2 r)_x + \mu_2(|r|^2)_x r], \end{aligned} \tag{2}$$

where a_j, b_j, c_j, d_j, e_j, f_j, γ_j, η_j, δ_j, λ_j and μ_j for $(j = 1, 2)$ are all real-valued constants and $i = \sqrt{-1}$, while the independent variables x and t represent spatial and temporal variables, respectively. The dependent variables $q = q(x, t)$ and $r = r(x, t)$ are the complex valued describing the wave profiles for the two components in polarization-preserving fibers. The first terms in Eqs. (1) and (2) are linear temporal evolution of the pulses. Next, the coefficients of a_j are the 3ODs and the coefficients b_j

stand for the 4ODs. Then, c_j are the self-phase modulation (SPM) terms while d_j stand for cross-phase modulation (XPM). Next, f_j represent nonlinear dispersions, while η_j are four wave-mixing (4WM) terms. The parameters e_j coupled with c_j give SPM and when coupled with d_j give XPM. Then γ_j give the XPM along with 4WM. Finally, δ_j are the coefficients of inter-modal dispersion (IMD) and the parameters λ_j give self-steepening terms (SSTs) that avoid the formation of shock waves, while μ_j are nonlinear dispersion coefficients.

This paper is organized as follows. In Sec. 2, the mathematical preliminaries are introduced. In Secs. 3 and 4, the generalized auxiliary equation method and the addendum to Kudryashov's method are successfully applied. In Sec. 6, the conservation laws of the model are addressed with a few conclusive words that are given in Sec. 7.

2. Mathematical Preliminaries

In order to solve Eqs. (1) and (2), we assume the hypothesis:

$$\begin{aligned} q(x,t) &= U_1(\xi)\exp[i\psi(x,t)], \\ r(x,t) &= U_2(\xi)\exp[i\psi(x,t)], \end{aligned} \tag{3}$$

such that

$$\xi = x - vt, \quad \psi(x,t) = -\kappa x + \omega t + \theta_0, \tag{4}$$

where v, κ, ω and θ_0 are all nonzero constants to be determined which represent soliton velocity, soliton frequency, wave number and phase constant, respectively. Next, $\psi(x,t)$ is a real function which represents the phase component of the soliton, while $U_j(\xi)$ are real functions which represent the shape of the pulse of the solitons. Substituting (3) along with (4) into Eqs. (1) and (2), separating the real and imaginary parts, we deduce that the real parts are

$$\begin{aligned} &b_1U_1'''' + (3a_1\kappa - 6b_1\kappa^2)U_1'' - [\omega + \delta_1\kappa + a_1\kappa^3 - b_1\kappa^4]U_1 \\ &\quad + (e_1 + f_1\kappa)(c_1U_1^2 + d_1U_2^2)U_1 + (\gamma_1 + \eta_1\kappa)U_1U_2^2 - \lambda_1\kappa U_1^3 = 0 \end{aligned} \tag{5}$$

and

$$\begin{aligned} &b_2U_2'''' + (3a_2\kappa - 6b_2\kappa^2)U_2'' - [\omega + \delta_2\kappa + a_2\kappa^3 - b_2\kappa^4]U_2 \\ &\quad + (e_2 + f_2\kappa)(c_2U_2^2 + d_2U_1^2)U_2 + (\gamma_2 + \eta_2\kappa)U_2U_1^2 - \lambda_2\kappa U_2^3 = 0, \end{aligned} \tag{6}$$

while the imaginary parts are

$$\begin{aligned} &(a_1 - 4b_1\kappa)U_1''' - (\delta_1 + v + 3a_1\kappa^2 - 4b_1\kappa^3)U_1' - (3\lambda_1 + 2\mu_1)U_1^2U_1' \\ &\quad + f_1(c_1U_1^2 + d_1U_2^2)U_1' + \eta_1U_1U_2U_2' = 0 \end{aligned} \tag{7}$$

and

$$\begin{aligned}(a_2 - 4b_2\kappa)U_2''' - (\delta_2 + v + 3a_2\kappa^2 - 4b_2\kappa^3)U_2' - (3\lambda_2 + 2\mu_2)U_2^2 U_2' \\ + f_2(c_2U_2^2 + d_2U_1^2)U_2' + \eta_2 U_2 U_1 U_1' = 0.\end{aligned} \tag{8}$$

Set

$$U_2(\xi) = \chi U_1(\xi), \tag{9}$$

where χ is a nonzero constant, such that $\chi \neq 1$. Consequently, the real parts change to

$$\begin{aligned}b_1U_1'''' + (3a_1\kappa - 6b_1\kappa^2)U_1'' - [\omega + \delta_1\kappa + a_1\kappa^3 - b_1\kappa^4]U_1 \\ + [(e_1 + f_1\kappa)(c_1 + d_1\chi^2) + (\gamma_1 + \eta_1\kappa)\chi^2 - \lambda_1\kappa]U_1^3 = 0\end{aligned} \tag{10}$$

and

$$\begin{aligned}\chi b_2U_1'''' + (3a_2\kappa - 6b_2\kappa^2)\chi U_1'' - [\omega + \delta_2\kappa + a_2\kappa^3 - b_2\kappa^4]\chi U_1 \\ + [(e_2 + f_2\kappa)(c_2\chi^2 + d_2)\chi + (\gamma_2 + \eta_2\kappa)\chi - \lambda_2\kappa\chi^3]U_1^3 = 0,\end{aligned} \tag{11}$$

while the imaginary parts become

$$\begin{aligned}(a_1 - 4b_1\kappa)U_1''' - (\delta_1 + v + 3a_1\kappa^2 - 4b_1\kappa^3)U_1' \\ + [f_1(c_1 + d_1\chi^2) - (3\lambda_1 + 2\mu_1) + \eta_1\chi^2]U_1^2U_1' = 0\end{aligned} \tag{12}$$

and

$$\begin{aligned}(a_2 - 4b_2\kappa)\chi U_1''' - (\delta_2 + v + 3a_2\kappa^2 - 4b_2\kappa^3)\chi U_1' \\ + [f_2(c_2\chi^2 + d_2)\chi - (3\lambda_2 + 2\mu_2)\chi^3 + \eta_2\chi]U_1^2U_1' = 0.\end{aligned} \tag{13}$$

The linearly independent principle is applied on (12) and (13) to get the frequency of soliton:

$$\kappa = \frac{a_j}{4b_j}, \quad a_j \neq 0,\ b_j \neq 0 \quad \text{for } j = 1, 2, \tag{14}$$

the velocity of soliton is given by

$$v = 4b_j\kappa^3 - \delta_j - 3a_j\kappa^2 \quad \text{for } j = 1, 2, \tag{15}$$

and the constraint conditions

$$\begin{aligned}f_1(c_1 + d_1\chi^2) - (3\lambda_1 + 2\mu_1) + \eta_1\chi^2 = 0, \\ f_2(c_2\chi^2 + d_2) - (3\lambda_2 + 2\mu_2)\chi^2 + \eta_2 = 0.\end{aligned} \tag{16}$$

From (14) and (15), one gets the natural constraint relations as

$$\begin{aligned}a_1b_2 = a_2b_1, \\ 4b_1\kappa^3 - \delta_1 - 3a_1\kappa^2 = 4b_2\kappa^3 - \delta_2 - 3a_2\kappa^2.\end{aligned} \tag{17}$$

Equations (10) and (11) have the same form under the constraint conditions:

$$\begin{aligned}
&b_1 = \chi b_2,\\
&3a_1\kappa - 6b_1\kappa^2 = (3a_2\kappa - 6b_2\kappa^2)\chi,\\
&\omega + \delta_1\kappa + a_1\kappa^3 - b_1\kappa^4 = (\omega + \delta_2\kappa + a_2\kappa^3 - b_2\kappa^4)\chi,\\
&(e_1 + f_1\kappa)(c_1 + d_1\chi^2) + (\gamma_1 + \eta_1\kappa)\chi^2 - \lambda_1\kappa\\
&\qquad = (e_2 + f_2\kappa)(c_2\chi^2 + d_2)\chi + (\gamma_2 + \eta_2\kappa)\chi - \lambda_2\kappa\chi^3.
\end{aligned} \tag{18}$$

From (18), we have the wave number of the soliton as

$$\omega = \frac{\delta_1\kappa + a_1\kappa^3 - b_1\kappa^4 - (\delta_2\kappa + a_2\kappa^3 - b_2\kappa^4)\chi}{(\chi - 1)}. \tag{19}$$

Equation (10) can be rewritten as

$$b_1 U_1'''' + \Pi_0 U_1'' + \Pi_1 U_1 + \Pi_3 U_1^3 = 0, \tag{20}$$

where

$$\left.\begin{aligned}
\Pi_0 &= 3a_1\kappa - 6b_1\kappa^2,\\
\Pi_1 &= -[\omega + \delta_1\kappa + a_1\kappa^3 - b_1\kappa^4],\\
\Pi_3 &= (e_1 + f_1\kappa)(c_1 + d_1\chi^2) + (\gamma_1 + \eta_1\kappa)\chi^2 - \lambda_1\kappa.
\end{aligned}\right\} \tag{21}$$

Now, we will solve Eq. (20) using the following methods.

3. The Generalized Auxiliary Equation Method

According to this method, we assume that Eq. (20) has the formal solution:

$$U_1(\xi) = \sum_{l=0}^{N} \alpha_l f^l(\xi), \tag{22}$$

where $f(\xi)$ satisfies the first-order auxiliary equation:

$$f'^2(\xi) = \sum_{m=0}^{6} h_m f^m(\xi). \tag{23}$$

Here, $\alpha_l (l = 0, 1, \ldots, N)$ and $h_m (m = 0, 1, \ldots, 6)$ are constants to be determined such that $\alpha_N \neq 0$ and $h_6 \neq 0$, where N is a positive integer. Balancing U_1'''' and U_1^3 in Eq. (20), one gets the balance number $N = 4$. From (22), one gets the formal solution of Eq. (20) as

$$U_1(\xi) = \alpha_0 + \alpha_1 f(\xi) + \alpha_2 f^2(\xi) + \alpha_3 f^3(\xi) + \alpha_4 f^4(\xi), \tag{24}$$

where $\alpha_l (l = 0, 1, 2, 3, 4)$ are constants to be determined such that $\alpha_4 \neq 0$. Substituting (24) and (23) with $h_0 = h_1 = h_3 = h_5 = 0$ into Eq. (20), collecting all the coefficients of $[f(\xi)]^{m_1}[f(\xi)]^{m_2} (m_1 = 0, 1, \ldots, 12, m_2 = 0, 1)$ and setting these coefficients to zero, we have a set of algebraic equations which can be solved using the

Maple to obtain the following results:

$$\left.\begin{array}{l}
\Pi_3\alpha_4^3+1920b_1\alpha_4h_6^2=0,\\
3\Pi_3\alpha_3\alpha_4^2+945b_1\alpha_3h_6^2=0,\\
3\Pi_3\alpha_2\alpha_4^2+384b_1\alpha_2h_6^2+3\Pi_3\alpha_3^2\alpha_4+2688b_1\alpha_4h_6h_4=0,\\
105b_1\alpha_1h_6^2+1260b_1h_4\alpha_3h_6+6\Pi_3\alpha_2\alpha_3\alpha_4+3\Pi_3\alpha_1\alpha_4^2+\Pi_3\alpha_3^3=0,\\
6\Pi_3\alpha_1\alpha_3\alpha_4+3\Pi_3\alpha_2\alpha_3^2+24\Pi_0\alpha_4h_6+3\Pi_3\alpha_0\alpha_4^2+1920b_1\alpha_4h_6h_2\\
\qquad+\ 840b_1\alpha_4h_4^2+3\Pi_3\alpha_2^2\alpha_4+480b_1h_4\alpha_2h_6=0,\\
870b_1h_6\alpha_3h_2+6\Pi_3\alpha_1\alpha_2\alpha_4+15\Pi_0\alpha_3h_6+3\Pi_3\alpha_1\alpha_3^2+360b_1\alpha_3h_4^2\\
\qquad+\ 6\Pi_3\alpha_0\alpha_3\alpha_4+120b_1h_4\alpha_1h_6+3\Pi_3\alpha_2^2\alpha_3=0,\\
320b_1h_6\alpha_2h_2+6\Pi_3\alpha_0\alpha_2\alpha_4+6\Pi_3\alpha_1\alpha_2\alpha_3+3\Pi_3\alpha_1^2\alpha_4+20\Pi_0\alpha_4h_4\\
\qquad+\ 8\Pi_0\alpha_2h_6+\Pi_3\alpha_2^3+120b_1\alpha_2h_4^2+3\Pi_3\alpha_0\alpha_3^2+1040b_1h_4\alpha_4h_2=0,\\
24b_1\alpha_1h_4^2+78b_1h_6\alpha_1h_2+6\Pi_3\alpha_0\alpha_2\alpha_3+6\Pi_3\alpha_0\alpha_1\alpha_4+408b_1h_4\alpha_3h_2\\
\qquad+\ 3\Pi_3\alpha_1\alpha_2^2+12\Pi_0\alpha_3h_4+3\Pi_0\alpha_1h_6+3\Pi_3\alpha_1^2\alpha_3=0,\\
120b_1h_4\alpha_2h_2+6\Pi_3\alpha_0\alpha_1\alpha_3+\Pi_1\alpha_4+6\Pi_0\alpha_2h_4+256b_1\alpha_4h_2^2\\
\qquad+\ 3\Pi_3\alpha_0^2\alpha_4+3\Pi_3\alpha_0\alpha_2^2+3\Pi_3\alpha_1^2\alpha_2+16\Pi_0\alpha_4h_2=0,\\
20b_1h_4\alpha_1h_2+9\Pi_0\alpha_3h_2+6\Pi_3\alpha_0\alpha_1\alpha_2+81b_1\alpha_3h_2^2+3\Pi_3\alpha_0^2\alpha_3\\
\qquad+\ \Pi_3\alpha_1^3+2\Pi_0\alpha_1h_4+\Pi_1\alpha_3=0,\\
3\Pi_3\alpha_0^2\alpha_2+16b_1\alpha_2h_2^2+3\Pi_3\alpha_0\alpha_1^2+4\Pi_0\alpha_2h_2+\Pi_1\alpha_2=0,\\
3\Pi_3\alpha_0^2\alpha_1+b_1\alpha_1h_2^2+\Pi_0\alpha_1h_2+\Pi_1\alpha_1=0,\\
\Pi_1\alpha_0+\Pi_3\alpha_0^3=0.
\end{array}\right\}\tag{25}$$

On solving the above algebraic equations (25) by using the Maple, one gets the following two results:

Result 1:

$$\begin{gathered}
\alpha_0=0,\quad \alpha_1=0,\ \alpha_2=4\epsilon\sqrt{\frac{6\Pi_0h_6}{\Pi_3}},\ \alpha_3=0,\ \alpha_4=8\epsilon h_6\sqrt{-\frac{30b_1}{\Pi_3}},\\
h_2=-\frac{\Pi_0}{20b_1},\ h_4=-\epsilon\sqrt{-\frac{\Pi_0h_6}{5b_1}},\ h_6=h_6
\end{gathered}\tag{26}$$

and

$$\Pi_1=\frac{4\Pi_0^2}{25b_1},\tag{27}$$

provided $\Pi_0\Pi_3h_6>0$, $\Pi_0b_1h_6<0$, $b_1\Pi_3<0$ and $\epsilon=\pm1$. It is well known that Eq. (23) has the following solutions: When $h_0=h_1=h_3=h_5=0$, we have

$$f(\xi)=\left[-\frac{h_2h_4\operatorname{sech}^2(\sqrt{h_2}\xi)}{h_4^2-h_2h_6[1+\epsilon\tanh(\sqrt{h_2}\xi)]^2}\right]^{\frac{1}{2}},\tag{28}$$

$$f(\xi)=\left[\frac{h_2h_4\mathrm{csch}^2(\sqrt{h_2}\xi)}{h_4^2-h_2h_6[1+\epsilon\coth(\sqrt{h_2}\xi)]^2}\right]^{\frac{1}{2}}, \tag{29}$$

provided $h_2>0$, $\epsilon=\pm1$,

$$f(\xi)=\left[-\frac{h_2\mathrm{sech}^2(\sqrt{h_2}\xi)}{h_4+2\epsilon\sqrt{h_2h_6}\tanh(\sqrt{h_2}\xi)}\right]^{\frac{1}{2}}, \tag{30}$$

$$f(\xi)=\left[\frac{h_2\mathrm{csch}^2(\sqrt{h_2}\xi)}{h_4+2\epsilon\sqrt{h_2h_6}\coth(\sqrt{h_2}\xi)}\right]^{\frac{1}{2}}, \tag{31}$$

provided $h_2>0, h_6>0$ and $\epsilon=\pm1$.

Substituting (26) along with (28)–(31) into (24), one gets the following soliton solutions of the system (1) and (2):

$$\begin{aligned} q(x,t)=&\pm\frac{4}{5}\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\frac{\mathrm{sech}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{4-\left[1+\epsilon\tanh\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)\right]^2}\\ &\left[1-\frac{2\mathrm{sech}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{4-\left[1+\epsilon\tanh\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)\right]^2}\right]e^{i(-\kappa x+\omega t+\theta_0)} \end{aligned} \tag{32}$$

and

$$\begin{aligned} r(x,t)=&\pm\frac{4}{5}\chi\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\frac{\mathrm{sech}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{4-\left[1+\epsilon\tanh\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)\right]^2}\\ &\left[1-\frac{2\mathrm{sech}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{4-\left[1+\epsilon\tanh\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)\right]^2}\right]e^{i(-\kappa x+\omega t+\theta_0)}, \end{aligned} \tag{33}$$

or

$$\begin{aligned} q(x,t)=&\pm\frac{4}{5}\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\frac{\mathrm{csch}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{4-\left[1+\epsilon\coth\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)\right]^2}\\ &\left[1+\frac{2\mathrm{csch}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{4-\left[1+\epsilon\coth\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)\right]^2}\right]e^{i(-\kappa x+\omega t+\theta_0)} \end{aligned} \tag{34}$$

and

$$r(x,t) = \pm\frac{4}{5}\chi\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\frac{\operatorname{csch}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{4-\left[1+\epsilon\coth\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)\right]^2}\left[1+\frac{2\operatorname{csch}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{4-\left[1+\epsilon\coth\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)\right]^2}\right]e^{i(-\kappa x+\omega t+\theta_0)}, \tag{35}$$

or

$$q(x,t) = \pm\frac{1}{2}\Pi_0\sqrt{-\frac{6}{5b_1\Pi_3}}\left[\frac{\operatorname{sech}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{1+\epsilon\tanh\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}\right]\left[2-\frac{\operatorname{sech}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{1+\epsilon\tanh\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}\right]e^{i(-\kappa x+\omega t+\theta_0)}, \tag{36}$$

and

$$r(x,t) = \pm\frac{1}{2}\chi\Pi_0\sqrt{-\frac{6}{5b_1\Pi_3}}\left[\frac{\operatorname{sech}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{1+\epsilon\tanh\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}\right]\left[2-\frac{\operatorname{sech}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{1+\epsilon\tanh\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}\right]e^{i(-\kappa x+\omega t+\theta_0)}, \tag{37}$$

or

$$q(x,t) = \pm\frac{1}{2}\Pi_0\sqrt{-\frac{6}{5b_1\Pi_3}}\left[\frac{\operatorname{csch}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{1+\epsilon\coth\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}\right]\left[2+\frac{\operatorname{csch}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{1+\epsilon\coth\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}\right]e^{i(-\kappa x+\omega t+\theta_0)}, \tag{38}$$

and

$$r(x,t) = \pm\frac{1}{2}\chi\Pi_0\sqrt{-\frac{6}{5b_1\Pi_3}}\left[\frac{\operatorname{csch}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{1+\epsilon\coth\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}\right]\left[2+\frac{\operatorname{csch}^2\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}{1+\epsilon\coth\left(\sqrt{-\frac{\Pi_0}{20b_1}}(x-vt)\right)}\right]e^{i(-\kappa x+\omega t+\theta_0)}, \tag{39}$$

provided $b_1\Pi_3 < 0$ and $b_1\Pi_0 < 0$.

The solutions (32)–(39) exist under the constraint condition (27).

Result 2:

$$\begin{gathered}\alpha_0 = 0, \quad \alpha_1 = 0,\ \alpha_2 = 0,\ \alpha_3 = 0,\ \alpha_4 = 8\epsilon h_6\sqrt{-\frac{30b_1}{\Pi_3}}, \\ h_2 = -\frac{\Pi_0}{80b_1}, \quad h_4 = 0,\ h_6 = h_6\end{gathered} \tag{40}$$

and

$$\Pi_1 = \frac{4\Pi_0^2}{25b_1}, \tag{41}$$

provided $b_1\Pi_3 < 0$ and $\epsilon = \pm 1$. It is well known that Eq. (23) has the following solutions:

When $h_0 = h_1 = h_3 = h_5 = 0$, we have

$$f(\xi) = \left[\frac{2h_2}{\epsilon\sqrt{h_4^2 - 4h_2h_6}\cosh\left(2\sqrt{h_2}\xi\right) - h_4}\right]^{\frac{1}{2}}, \tag{42}$$

provided $h_2 > 0, h_4^2 - 4h_2h_6 > 0$ and $\epsilon = \pm 1$,

$$f(\xi) = \left[\frac{2h_2}{\epsilon\sqrt{-\left(h_4^2 - 4h_2h_6\right)}\sinh\left(2\sqrt{h_2}\xi\right) - h_4}\right]^{\frac{1}{2}}, \tag{43}$$

provided $h_2 > 0, h_4^2 - 4h_2h_6 < 0$ and $\epsilon = \pm 1$,

$$f(\xi) = \left[\frac{2h_2}{\epsilon\sqrt{h_4^2 - 4h_2h_6}\cos\left(2\sqrt{-h_2}\xi\right) - h_4}\right]^{\frac{1}{2}}, \tag{44}$$

$$f(\xi) = \left[\frac{2h_2}{\epsilon\sqrt{h_4^2 - 4h_2h_6}\sin\left(2\sqrt{-h_2}\xi\right) - h_4}\right]^{\frac{1}{2}}, \tag{45}$$

provided $h_2 < 0, h_4^2 - 4h_2h_6 > 0$ and $\epsilon = \pm 1$.

Substituting (40) along with (42)–(45) into (24), one gets the following solutions of the system (1) and (2):

(I) The bright soliton solutions:

$$q(x,t) = \pm\frac{1}{10}\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\operatorname{sech}^2\left(\frac{1}{2}\sqrt{-\frac{\Pi_0}{5b_1}}(x - vt)\right)e^{i(-\kappa x + \omega t + \theta_0)} \tag{46}$$

and

$$r(x,t) = \pm\frac{1}{10}\chi\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\operatorname{sech}^2\left(\frac{1}{2}\sqrt{-\frac{\Pi_0}{5b_1}}(x - vt)\right)e^{i(-\kappa x + \omega t + \theta_0)}, \tag{47}$$

provided $b_1\Pi_3 < 0$ and $b_1\Pi_0 < 0$.

(II) The singular soliton solutions:

$$q(x,t) = \pm\frac{1}{10}\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\operatorname{csch}^2\left(\frac{1}{2}\sqrt{-\frac{\Pi_0}{5b_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)} \tag{48}$$

and

$$r(x,t) = \pm\frac{1}{10}\chi\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\operatorname{csch}^2\left(\frac{1}{2}\sqrt{-\frac{\Pi_0}{5b_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}, \tag{49}$$

provided $b_1\Pi_3 < 0$ and $b_1\Pi_0 < 0$.

(III) The periodic wave solutions:

$$q(x,t) = \pm\frac{1}{10}\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\sec^2\left(\frac{1}{2}\sqrt{\frac{\Pi_0}{5b_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)} \tag{50}$$

and

$$r(x,t) = \pm\frac{1}{10}\chi\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\sec^2\left(\frac{1}{2}\sqrt{\frac{\Pi_0}{5b_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}, \tag{51}$$

or

$$q(x,t) = \pm\frac{1}{10}\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\csc^2\left(\frac{1}{2}\sqrt{\frac{\Pi_0}{5b_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)} \tag{52}$$

and

$$r(x,t) = \pm\frac{1}{10}\chi\Pi_0\sqrt{-\frac{30}{b_1\Pi_3}}\csc^2\left(\frac{1}{2}\sqrt{\frac{\Pi_0}{5b_1}}(x-vt)\right)e^{i(-\kappa x+\omega t+\theta_0)}, \tag{53}$$

provided $b_1\Pi_3 < 0$ and $b_1\Pi_0 > 0$.

The solutions (46)–(53) exist under the constraint condition (41).

4. An Addendum to Kudryashov's Method

According to this method, we assume that Eq. (20) has the formal solution

$$U_1(\xi) = \sum_{l=0}^{N} B_l R^l(\xi), \tag{54}$$

where $B_l(l = 0, 1, 2, \dots, N)$ are constants to be determined, such that $B_N \neq 0$, while $R(\xi)$ satisfies the first-order auxiliary ordinary differential equation (ODE):

$$R'^2(\xi) = R^2(\xi)[1 - \Gamma R^{2p}(\xi)]\ln^2 K, \quad 0 < K \neq 1, \tag{55}$$

where Γ is a constant. It is easy to show that Eq. (55) has the following solution:

$$R(\xi) = \left[\frac{4A}{4A^2 \exp_K(p\xi) + \Gamma \exp_K(-p\xi)}\right]^{\frac{1}{p}}, \tag{56}$$

where A is a nonzero constant, p is a positive integer and $\exp_K(p\xi) = K^{p\xi}$. We determine the positive integer N in (54) by using the homogeneous balance method as follows.

If $D(U_1) = N, D(U_1') = N + p, D(U_1'') = N + 2p$, then we have $D[U_1^r U_1^{(s)}] = N(r+1) + ps$.

Now, by balancing U_1'''' with U_1^3 in Eq. (20), one gets

$$N + 4p = 3N \Rightarrow N = 2p. \tag{57}$$

Next, we will discuss the following cases.

Case 1. If we choose $p = 1$, then $N = 2$. Thus, from (54) we deduce that Eq. (20) has the formal solution:

$$U_1(\xi) = B_0 + B_1 R(\xi) + B_2 R^2(\xi), \tag{58}$$

where B_0, B_1 and B_2 are constants to be determined, such that $B_2 \neq 0$ while $R(\xi)$ satisfies the first-order auxiliary ODE:

$$R'^2(\xi) = R^2(\xi)[1 - \Gamma R^2(\xi)]\ln^2 K, \quad 0 < K \neq 1, \tag{59}$$

where Γ is a constant. Substituting (58) along with (59) into Eq. (20), collecting all the coefficients of each power of $[R(\xi)]^{m_1}[R'(\xi)]^{m_2}(m_1 = 0, 1, \ldots, 6, m_2 = 0, 1)$ and setting each of these coefficients to zero, we have a system of algebraic equations which can be solved by using the Maple to get the results:

$$B_0 = 0, \quad B_1 = 0, \; B_2 = \pm 2\Gamma\sqrt{-\frac{30b_1}{\Pi_3}}\ln^2 K \tag{60}$$

and

$$\Pi_0 = -20b_1 \ln^2 K, \quad \Pi_1 = 64b_1 \ln^4 K, \tag{61}$$

provided $b_1\Pi_3 < 0$. Substituting (60) along with (56) into Eq. (58), one gets the solutions of the system (1) and (2) in the forms:

$$q(x,t) = \pm 2\Gamma\sqrt{-\frac{30b_1}{\Pi_3}}\left[\frac{4A\ln K}{4A^2 K^{(x-ct)} + \Gamma K^{-(x-ct)}}\right]^2 e^{i(-\kappa x + \omega t + \theta_0)} \tag{62}$$

and

$$r(x,t) = \pm 2\chi\Gamma\sqrt{-\frac{30b_1}{\Pi_3}}\left[\frac{4A\ln K}{4A^2 K^{(x-ct)} + \Gamma K^{-(x-ct)}}\right]^2 e^{i(-\kappa x + \omega t + \theta_0)}. \tag{63}$$

In particular, if we set $\Gamma = 4A^2$ in (62) and (63), then the system (1) and (2) has the bright soliton solutions as

$$q(x,t) = \pm 2\sqrt{-\frac{30b_1}{\Pi_3}}(\ln^2 K)\operatorname{sech}^2[(x-ct)\ln K]e^{i(-\kappa x+\omega t+\theta_0)} \tag{64}$$

and

$$r(x,t) = \pm 2\chi\sqrt{-\frac{30b_1}{\Pi_3}}(\ln^2 K)\operatorname{sech}^2[(x-ct)\ln K]e^{i(-\kappa x+\omega t+\theta_0)}, \tag{65}$$

while, if $\Gamma = -4A^2$, one gets the singular soliton solution of the system (1) and (2) as

$$q(x,t) = \pm 2\sqrt{-\frac{30b_1}{\Pi_3}}(\ln^2 K)\operatorname{csch}^2[(x-ct)\ln K]e^{i(-\kappa x+\omega t+\theta_0)} \tag{66}$$

and

$$r(x,t) = \pm 2\chi\sqrt{-\frac{30b_1}{\Pi_3}}(\ln^2 K)\operatorname{csch}^2[(x-ct)\ln K]e^{i(-\kappa x+\omega t+\theta_0)}. \tag{67}$$

The solutions (62)–(67) exist under the conditions (61).

Case 2. If we choose $p = 2$, then $N = 4$. Thus, we deduce from (54) that Eq. (20) has the formal solution:

$$U_1(\xi) = B_0 + B_1R(\xi) + B_2R^2(\xi) + B_3R^3(\xi) + B_4R^4(\xi), \tag{68}$$

where B_0, B_1, B_2, B_3 and B_4 are constants to be determined, such that $B_4 \neq 0$ and the function $R(\xi)$ satisfies the first-order auxiliary ODE

$$R'^2(\xi) = R^2(\xi)[1 - \Gamma R^4(\xi)]\ln^2 K, \quad 0 < K \neq 1, \tag{69}$$

where Γ is a constant. Substituting (68) along with (69) into Eq. (20), collecting all the coefficients of each power of $[R(\xi)]^{m_1}[R'(\xi)]^{m_2}(m_1 = 0,1,2,\ldots,12, m_2 = 0,1)$ and setting each of these coefficients to zero, we have a system of algebraic equations which can be solved by using the Maple to get the results:

$$B_0 = 0, \quad B_1 = 0,\ B_2 = 0,\ B_3 = 0,\ B_4 = \pm 8\Gamma\sqrt{-\frac{30b_1}{\Pi_3}}\ln^2 K \tag{70}$$

and

$$\Pi_0 = -80b_1\ln^2 K, \quad \Pi_1 = 1024b_1\ln^4 K, \tag{71}$$

provided $b_1\Pi_3 < 0$. Substituting (70) along with (56) into Eq. (68), one gets the solutions of the system (1) and (2) in the forms:

$$q(x,t) = \pm 8\Gamma\sqrt{-\frac{30b_1}{\Pi_3}}\left[\frac{4A\ln K}{4A^2K^{2(x-ct)} + \Gamma K^{-2(x-ct)}}\right]^2 e^{i(-\kappa x+\omega t+\theta_0)} \tag{72}$$

and

$$r(x,t) = \pm 8\chi\Gamma\sqrt{-\frac{30b_1}{\Pi_3}}\left[\frac{4A\ln K}{4A^2K^{2(x-ct)} + \Gamma K^{-2(x-ct)}}\right]^2 e^{i(-\kappa x+\omega t+\theta_0)}. \tag{73}$$

In particular, if we set $\Gamma = 4A^2$ in (72) and (73), then we have the bright soliton solution of the system (1) and (2) as

$$q(x,t) = \pm 8\sqrt{-\frac{30b_1}{\Pi_3}}(\ln^2 K)\operatorname{sech}^2[2(x-ct)\ln K]e^{i(-\kappa x+\omega t+\theta_0)} \tag{74}$$

and

$$r(x,t) = \pm 8\chi\sqrt{-\frac{30b_1}{\Pi_3}}(\ln^2 K)\operatorname{sech}^2[2(x-ct)\ln K]e^{i(-\kappa x+\omega t+\theta_0)}, \tag{75}$$

while, if $\Gamma = -4A^2$, one gets the singular soliton solution of the system (1) and (2) as

$$q(x,t) = \pm 8\sqrt{-\frac{30b_1}{\Pi_3}}(\ln^2 K)\operatorname{csch}^2[2(x-ct)\ln K]e^{i(-\kappa x+\omega t+\theta_0)} \tag{76}$$

and

$$r(x,t) = \pm 8\chi\sqrt{-\frac{30b_1}{\Pi_3}}(\ln^2 K)\operatorname{csch}^2[2(x-ct)\ln K]e^{i(-\kappa x+\omega t+\theta_0)}. \tag{77}$$

The solutions (72)–(77) exist under the conditions (71).

Similarly, we can find many other solutions by choosing other values for p and N.

5. Conservation Laws

Following some detailed calculations, we get that system (1) and (2) admits three conservation laws corresponding to "power", "linear momentum" and "Hamiltonian" for particular cases of the parameters; the conserved densities are, respectively, given as follows:

(i) If $\eta_1 = \eta_2$ and $d_1 = d_2 = 0$, we get the conserved "power density"

$$T_c^t = \frac{1}{2}(|q|^2 + |r|^2). \tag{78}$$

(ii) If $\eta_1 = \eta_2$, $d_1 = d_2 = 0$, $\lambda_1 = -\mu_2$, $\lambda_2 = -\mu_2$ and $\gamma_1 = \gamma_2$, the conserved "linear momentum density" is

$$T_m^t = \frac{1}{2}(\mathcal{I}(q^*q_x) + \mathcal{I}(r^*r_x)). \tag{79}$$

(iii) If, in addition to the conditions in (ii), $\eta_1 = \eta_2 = 0$, the conserved "Hamiltonian density" is

$$\begin{aligned} T_e^t = & -a_1\mathcal{I}(q^*q_{xxx}) - a_2\mathcal{I}(r^*r_{xxx}) + b_1\mathcal{R}(qq^*_{xxxx}) + b_2\mathcal{R}(rr^*_{xxxx}) \\ & + \frac{1}{4}c_1|q|^2(e_1|q|^2 - f_1\mathcal{I}(q^*q_x)) + \frac{1}{4}c_2|r|^2(e_2|r|^2 - f_2\mathcal{I}(r^*r_x)) \\ & + \frac{1}{2}\delta_1\mathcal{I}(q^*q_x) + \frac{1}{2}\delta_2\mathcal{I}(r^*r_x) - \frac{1}{4}\mu_1|q|^2\mathcal{I}(q^*q_x) - \frac{1}{4}\mu_2|r|^2\mathcal{I}(r^*r_x) \\ & + \frac{1}{2}\gamma_2|q|^2|r|^2. \end{aligned} \tag{80}$$

The two-component bright soliton solutions recovered in this paper are structured as

$$q(x,t) = A_1\text{sech}^2[B(x - vt)]e^{i(-\kappa x + \omega t + \theta_0)}, \tag{81}$$

$$r(x,t) = A_2\text{sech}^2[B(x - vt)]e^{i(-\kappa x + \omega t + \theta_0)}. \tag{82}$$

Here, A_j $(j = 1, 2)$ are the amplitudes of the two solitons while B represents their inverse widths. Therefore, the conserved quantities are

$$P = \frac{1}{2}\int_{-\infty}^{\infty}(|q|^2 + |r|^2)dx = \frac{2}{3B}(|A_1|^2 + |A_2|^2), \tag{83}$$

$$M = \frac{1}{4i}\int_{-\infty}^{\infty}\{(q^*q_x - qq^*_x) + (r^*r_x - rr^*_x)\}dx = -\frac{2\kappa}{3B}(|A_1|^2 + |A_2|^2) \tag{84}$$

and

$$\begin{aligned} H = & \int_{-\infty}^{\infty}\left[-\frac{1}{2i}\{a_1(q^*q_{xxx} - qq^*_{xxx}) + a_2(r^*r_{xxx} - rr^*_{xxx})\}\right. \\ & + \frac{1}{2}\{b_1(q^*q_{xxxx} + qq^*_{xxxx}) + b_2(r^*r_{xxxx} + rr^*_{xxxx})\} \\ & - \frac{1}{8i}\{(c_1f_1 + \mu_1)|q|^2(q^*q_x - qq^*_x) + (c_2f_2 + \mu_2)|r|^2(r^*r_x - rr^*_x)\} \\ & \left. + \frac{1}{4i}\{\delta_1(q^*q_x - qq^*_x) + \delta_2(r^*r_x - rr^*_x)\} + \frac{\gamma_2}{2}|q|^2|r|^2\right]dx \\ = & -\frac{4\kappa}{15}(a_1A_1^2 + a_2A_2^2)(5\kappa^2 + 12B^2) + \frac{64}{21}B^5(b_1A_1^2 + b_2A_2^2) \\ & + \frac{224}{35}\kappa^2B(b_1A_1^4 + b_2A_2^4) + \frac{4\kappa^4}{3B}(b_1A_1^5 + b_2A_2^5) \\ & + \frac{8\kappa}{35B}\{(c_1f_1 + \mu_1)A_1^4 + (c_2f_2 + \mu_2)A_2^4\} - \frac{2\kappa}{3B}(\delta_1A_1^2 + \delta_2A_2^2) + \frac{16\gamma_2A_1^2A_2^2}{35B}. \end{aligned} \tag{85}$$

These conserved quantities are power (P), linear momentum (M) and Hamiltonian (H), respectively.

6. Conclusions

Today's work is a display of polarized soliton solutions and their conservation laws with FLE in birefringent fibers. Two integration algorithms have made this retrieval of soliton solutions possible while the approach of multipliers has successfully recovered the conservation laws. The conserved quantities are finally computed and exhibited by utilizing the soliton solutions that are recovered in the paper. These recovered and enumerated results serve as a gateway to extend the study further along in this direction. An immediate example would be to extend the model to address DWDM topology with FLE, studying the cascaded system and Thirring solitons. More results, aligned with recently reported works, will appear over time sequentially.[28–39] Consequently, the aspects of ghost pulses and suppression of intrachannel collision of solitons, stochastic perturbation of solitons are all on the table. These projects will be taken up and their results would be disseminated.

Disclosure

The authors also declare that there is no conict of interest.

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