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Highly dispersive optical solitons with a polynomial law of refractive index by Laplace–Adomian decomposition

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Abstract
This paper presents a numerical study of highly dispersive optical solitons that maintain a cubic–quintic–septic nonlinear (also know as polynomial) form of the refractive index. The Laplace–Adomian decomposition scheme is applied as a numerical algorithm to put the model into perspective. Both bright and dark soliton solutions are studied in this context. Both surface plots and contour plots of such solitons are presented. The error plots are also shown, demonstrating extremely low error measure values.

Keywords Nonlinear Schrödinger's equation · Cubic–quintic–septic law · Highly dispersive solitons · Laplace–Adomian decomposition method

1 Introduction

The concept of highly dispersive (HD) optical solitons emerged in 2019. There are four forms of nonlinear refractive index structures that support soliton solutions, viz. the Kerr law, quadratic–cubic law, polynomial law (also known as cubic–quintic–septic law), and nonlocal law. These laws have been studied extensively in the context of HD solitons for both polarization-preserving as well as birefringent fibers [1–11]. The soliton solutions are extracted and the conservation laws recovered for both polarization-preserving and birefringent fibers. Moreover, numerical studies on HD solitons have also been conducted for the Kerr and quadratic–cubic laws of nonlinearity. In this context, both bright and dark soliton solutions have been investigated, and studied numerically with impressively small error measure values. The current work moves a step forward to study HD solitons with the polynomial law of nonlinearity. The Laplace–Adomian decomposition method (LADM) is adopted as the numerical algorithm in this paper [12, 13]. Both bright and dark solitons are studied. The surface plots and contour plots are sketched, along with their respective error measures. These are all enumerated in the rest of the paper after a quick and brief summary of the governing model.

2 A description of the governing equation

The nonlinear Schrödinger equation (NLSE) for highly dispersive optical solitons with CQS nonlinearity can be expressed as [14–16]

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$$\mathrm{i}q_t + \mathrm{i}a_1 q_x + a_2 q_{xx} + \mathrm{i}a_3 q_{xxx} + a_4 q_{xxxx} + \mathrm{i}a_5 q_{xxxxx} + a_6 q_{xxxxxx} + \left(b_1|q|^2 + b_2|q|^4 + b_3|q|^6\right)q = 0, \tag{1}$$

with $q = q(x,t)$ denoting the wave profile, a complex-valued function depending on the spatial and temporal variables x and t. In Eq. (1), the first term stands for the linear temporal evolution while the a_j for $\le j \le 6$ are the coefficients corresponding to intermodal dispersion (IMD), group velocity dispersion (GVD), and third- (3OD), fourth- (4OD), fifth- (5OD), and sixth-order dispersion (6OD), respectively. Also in Eq. (1), the last coefficients b_1, b_2, and b_3 correspond to the cubic–quintic–septic law of nonlinearity. These coefficients are all real numbers, while $\mathrm{i} = \sqrt{-1}$.

This purely theoretical model was proposed in 2019 [14–16]. The highly dispersive effects stem from the six dispersion terms indicated above. The polynomial law of nonlinearity provides three nonlinear terms. A necessary delicate balance emerges between the dispersion and nonlinear terms that stem from the six dispersive and three nonlinear effects. No such materials with a polynomial law of nonlinearity and exhibiting such HD effects are yet known, since it was only proposed in 2019. moreover, note that the soliton propagation dynamics for such a model can only be achieved using eye diagrams at this stage. Synthesis and experimental work on such materials have yet to be carried out.

2.1 Highly dispersive soliton solutions

The concept of HD solitons first emerged in 2019 [14–16]. In Eq. (1), the presence of six dispersion terms will lead to a dominant effect of dispersion, leading to the term “highly dispersive” solitons. The other factor is that the presence of six dispersive effects will lead to a considerable slowdown of the solitons. However, this effect is neglected in the problem addressed herein, to focus on the numerical aspects. Moreover, a substantial amount of soliton radiation would be produced, a detrimental effect that has also been overlooked. No studies have yet been carried out on the radiative effects originating from such HD solitons.

The bright highly dispersive optical soliton solutions to Eq. (1) were recently obtained [15], having the general form

$$q(x,t) = A[\mathrm{sech}(x - vt)] \times \exp\{\mathrm{i}(-\kappa x + \omega t + \Omega)\}, \tag{2}$$

where A is the amplitude of the soliton and v, κ, ω, and Ω signify the speed of the soliton, the frequency, the wavenumber, and the phase constant, respectively.

The parameters v and ω can be given in terms of the coefficients of the model represented by Eq. (1) as follows:

$$v = a_1 - 2a_2\kappa - 8a_4\kappa^3 - 16a_5\kappa^4, \tag{3}$$

$$\omega = \frac{24a_1\kappa^2 - 4a_5\left(5\kappa^6 + 105\kappa^4 + 259\kappa^2\right) - 12A^2b_1\kappa\left(\kappa^2 - 1\right) - A^4b_2\kappa\left(3\kappa^4 + 10\kappa^2 - 9\right) + 900a_5}{24\kappa}, \tag{4}$$

while A, κ, and Ω are arbitrary constants.

Also, the dark highly dispersive optical soliton solutions were also reported [15], having the general form

$$q(x,t) = B[\tanh(x - vt)] \times \exp\{\mathrm{i}(-\kappa x + \omega t + \Omega)\}. \tag{5}$$

The parameters v and ω can be given in terms of the coefficients of the model represented by Eq. (1) as follows:

$$v = a_1 - 2a_2\kappa - 8a_4\kappa^3 - 16a_5\kappa^4, \tag{6}$$

$$\omega = \frac{6\kappa\left(a_4\left(3\kappa^4 - 20\kappa^2 - 24\right) + a_1\kappa\right) + 4a_5\left(10\kappa^6 - 75\kappa^4 + 64\kappa^2 + 240\right) + 3B^2b_1\kappa\left(\kappa^2 + 2\right)}{6\kappa}, \tag{7}$$

while B, κ, and Ω are arbitrary constants.

Note that the effects of higher-order nonlinear terms are visible in the wavenumber, as seen in Eqs. (4) and (7) for bright and dark solitons, respectively. Additionally, for bright solitons, it is well known that the effect of higher-order nonlinearities is felt in the structures of the soliton amplitude and its inverse width [17, 18]. These effects are embedded in the numerical simulations of the current work.

3 The application of the LADM

This section describes the general procedure for a numerical treatment of Eq. (1), with given initial conditions for dark and bright solitons. The method applied here is known as the Laplace–Adomian decomposition method (LADM), originally established in Refs. [19, 20].

Assuming that the solution of Eq. (1) can be decoupled into its real and imaginary parts, *i.e.* $q(x,t) = \mathsf{u}(x,t) + \mathrm{i}\mathsf{v}(x,t)$, we have

$$\begin{aligned} u_t = & - a_1 u_x - a_2 v_{xx} - a_3 u_{xxx} - a_4 v_{xxxx} \\ & - a_5 u_{xxxxx} - a_6 v_{xxxxxx} + N_1(u, v) \end{aligned} \tag{8}$$

$$\begin{aligned} v_t = & - a_1 v_x + a_2 u_{xx} - a_3 v_{xxx} + a_4 u_{xxxx} \\ & - a_5 v_{xxxxx} + a_6 u_{xxxxxx} + N_2(u, v) \end{aligned} \tag{9}$$

where N_1 and N_2 are the nonlinear terms, given in terms of their real and imaginary parts by

$$N_1(u, v) = u\left[b_1\left(u^2 + v^2\right) + b_2\left(u^2 + v^2\right)^2 + b_3\left(u^2 + v^2\right)^3\right] \tag{10}$$

and

$$N_2(u, v) = v\left[b_1\left(u^2 + v^2\right) + b_2\left(u^2 + v^2\right)^2 + b_3\left(u^2 + v^2\right)^3\right]. \tag{11}$$

Next, we apply the LADM to solve the system of differential equations (8)–(9) written in the form

$$\begin{aligned} D_t u = & - a_1 D_x^1 u - a_2 D_x^2 v - a_3 D_x^3 u - a_4 D_x^4 v \\ & - a_5 D_x^5 u - a_6 D_x^6 v + N_1(u, v), \end{aligned} \tag{12}$$

$$\begin{aligned} D_t v = & - a_1 D_x^1 v + a_2 D_x^2 u - a_3 D_x^3 v + a_4 D_x^4 u \\ & - a_5 D_x^5 v + a_6 D_x^6 u + N_2(u, v), \end{aligned} \tag{13}$$

considering the initial conditions

$$u(x, 0) = \mathfrak{Re}(q(x, 0)) \quad \text{and} \quad v(x, 0) = \mathfrak{Im}(q(x, 0)).$$

In the above system of differential equations, D_t denotes a temporal derivative while D_x^j signifies a spatial derivative of jth order.

The procedure of the method follows the following steps:

Step 1: Apply the temporal Laplace transform to both sides of Eqs. (12) and (13) to obtain

$$\begin{aligned} \mathscr{L}\{D_t u\} = & \mathscr{L}\{-a_1 D_x^1 u - a_2 D_x^2 v - a_3 D_x^3 u \\ & - a_4 D_x^4 v - a_5 D_x^5 u \\ & - a_6 D_x^6 v + N_1(u, v)\}, \end{aligned} \tag{14}$$

$$\begin{aligned} \mathscr{L}\{D_t v\} = & \mathscr{L}\{-a_1 D_x^1 v \\ & + a_2 D_x^2 u - a_3 D_x^3 v \\ & + a_4 D_x^4 u - a_5 D_x^5 v + a_6 D_x^6 u + N_2(u, v)\}. \end{aligned} \tag{15}$$

Using the differentiation property of the Laplace transform $\mathscr{L}$ and the initial conditions, we obtain

$$\begin{aligned} u(x, s) = & \frac{u(x, 0)}{s} + \frac{1}{s}\mathscr{L}\{-a_1 D_x^1 u - a_2 D_x^2 v \\ & - a_3 D_x^3 u - a_4 D_x^4 v \\ & - a_5 D_x^5 u - a_6 D_x^6 v + N_1(u, v)\}, \end{aligned} \tag{16}$$

$$\begin{aligned} v(x, s) = & \frac{v(x, 0)}{s} + \frac{1}{s}\mathscr{L} \\ & \{-a_1 D_x^1 v + a_2 D_x^2 u - a_3 D_x^3 v \\ & + a_4 D_x^4 u - a_5 D_x^5 v + a_6 D_x^6 u + N_2(u, v)\}. \end{aligned} \tag{17}$$

Step 2: The LADM consists of decomposing the components u and v of the solution as follows:

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t), \qquad v(x, t) = \sum_{n=0}^{\infty} v_n(x, t). \tag{18}$$

In addition, the nonlinear terms N_1 and N_2 are represented by an infinite series of so-called Adomian polynomials in two variables,

$$\begin{aligned} N_1(u, v) = & u\left[b_1\left(u^2 + v^2\right) + b_2\left(u^2 + v^2\right)^2 + b_3\left(u^2 + v^2\right)^3\right] \\ = & \sum_{n=0}^{\infty} A_n(u_0, u_1, \dots, u_n; v_0, v_1, \dots, v_n) \end{aligned} \tag{19}$$

and

$$\begin{aligned} N_2(u, v) = & v\left[b_1\left(u^2 + v^2\right) + b_2\left(u^2 + v^2\right)^2 + b_3\left(u^2 + v^2\right)^3\right] \\ = & \sum_{n=0}^{\infty} B_n(u_0, u_1, \dots, u_n; v_0, v_1, \dots, v_n) \end{aligned} \tag{20}$$

The Adomian polynomials A_n and B_n in two variables can be obtained for all forms of nonlinearity, being calculated by the following formulas [21]:

$$\begin{aligned} & A_n(u_0, \dots, u_n; v_0, \dots, v_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n} \\ & \left[N_1\left(\sum_{i=1}^{\infty} \lambda^i u_i; \sum_{i=1}^{\infty} \lambda^i v_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, \dots \end{aligned} \tag{21}$$

$$\begin{aligned} & B_n(u_0, \dots, u_n; v_0, \dots, v_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n} \\ & \left[N_2\left(\sum_{i=1}^{\infty} \lambda^i u_i; \sum_{i=1}^{\infty} \lambda^i v_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, \dots \end{aligned} \tag{22}$$

Substituting Eqs. (18), (19), and (20) into Eqs. (16) and (17) gives

$$\sum_{n=0}^{\infty} u_n(x,s) = \frac{u(x,0)}{s} + \frac{1}{s}\mathscr{L} \left\{ -(a_1D_x^1 + a_3D_x^3 + a_5D_x^5)\sum_{n=0}^{\infty} u_n - (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)\sum_{n=0}^{\infty} v_n + \sum_{n=0}^{\infty} A_n \right\}, \tag{23}$$

$$\sum_{n=0}^{\infty} v_n(x,s) = \frac{v(x,0)}{s} + \frac{1}{s}\mathscr{L} \left\{ -(a_1D_x^1 + a_3D_x^3 + a_3D_x^5)\sum_{n=0}^{\infty} v_n + (a_2D_x^2 + a_4D_x^4 + a_6D_x^6)\sum_{n=0}^{\infty} u_n + \sum_{n=0}^{\infty} B_n \right\}. \tag{24}$$

Applying the linearity of the Laplace transform in Eqs. (23) and (24) and comparing both sides of the coupled equations, we obtain the following recursive formula:

$$\begin{cases} u_0(x,s) = \frac{u(x,0)}{s} = \frac{1}{s}\mathfrak{Re}(q(x,0)) \\ v_0(x,s) = \frac{v(x,0)}{s} = \frac{1}{s}\mathfrak{Im}(q(x,0)), \end{cases} \tag{25}$$

$$\begin{cases} u_1(x,s) = \frac{1}{s}\mathscr{L}\{-\left(a_1D_x^1 + a_3D_x^3 + a_5D_x^5\right)u_0 - \left(a_2D_x^2 + a_4D_x^4 + a_6D_x^6\right)v_0 + A_0\} \\ v_1(x,s) = \frac{1}{s}\mathscr{L}\{-\left(a_1D_x^1 + a_3D_x^3 + a_5D_x^5\right)v_0 - \left(a_2D_x^2 + a_4D_x^4 + a_6D_x^6\right)u_0 + B_0\}, \\ \vdots \end{cases} \tag{26}$$

In general, for $m \geq 1$, we obtain

$$\begin{cases} u_{m+1}(x,s) = \frac{1}{s}\mathscr{L}\{-\left(a_1D_x^1 + a_3D_x^3 + a_5D_x^5\right)u_m - \left(a_2D_x^2 + a_4D_x^4 + a_6D_x^6\right)v_m + A_m\} \\ v_{m+1}(x,s) = \frac{1}{s}\mathscr{L}\{-\left(a_1D_x^1 + a_3D_x^3 + a_5D_x^5\right)v_m - \left(a_2D_x^2 + a_4D_x^4 + a_6D_x^6\right)u_m + B_m\}. \end{cases} \tag{27}$$

Step 3: Applying the inverse Laplace transform $\mathscr{L}^{-1}$ and the initial conditions to system given by Eqs. (25) and (27), we can evaluate u_m and v_m. Finally, the solution obtained by using the proposed method is in the form of infinite series, being given by

$$\begin{aligned} u(x,t) &= u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots \\ &= \sum_{n=0}^{\infty} u_n(x,t), \\ v(x,t) &= v_0(x,t) + v_1(x,t) + v_2(x,t) + \cdots \\ &= \sum_{n=0}^{\infty} v_n(x,t). \end{aligned} \tag{28}$$

The obtained series solution may converge to an exact solution if such a solution exists. Otherwise, the series solution can be used for numerical purposes. For further details regarding the convergence of the proposed method, refer to Refs. [22, 23].

In the next section, some numerical examples are given to illustrate the high accuracy and efficiency of the algorithm provided by the proposed method.

Table 1 The coefficients of Eq. (1) for bright highly dispersive optical solitons

Case	a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	κ	ν	N	| Max error |
1	1.50	0.45	0.85	0.60	1.05	1.25	0.85	0.05	0.34	0.50	0.40	15	4.0×10^{-8}
2	1.00	−0.25	0.75	0.33	0.15	1.01	0.20	0.01	0.95	1.00	−3.54	15	8.0×10^{-7}
3	1.40	0.01	0.22	0.15	−0.10	0.75	2.85	1.00	0.44	0.25	1.39	15	1.0×10^{-7}

Table 2 The coefficients of Eq. (1) for dark highly dispersive optical solitons

Cases	a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	κ	ν	N	| Max error |
4	2.10	0.25	1.05	0.16	2.05	−1.02	0.55	0.22	0.86	0.20	1.78	15	8.0×10^{-7}
5	1.39	0.34	0.96	0.66	0.18	1.55	1.89	0.05	0.34	0.14	2.09	15	1.5×10^{-7}
6	1.05	0.04	0.72	0.55	−0.30	−0.55	2.22	3.01	0.79	1.55	−2.95	15	2.0×10^{-7}

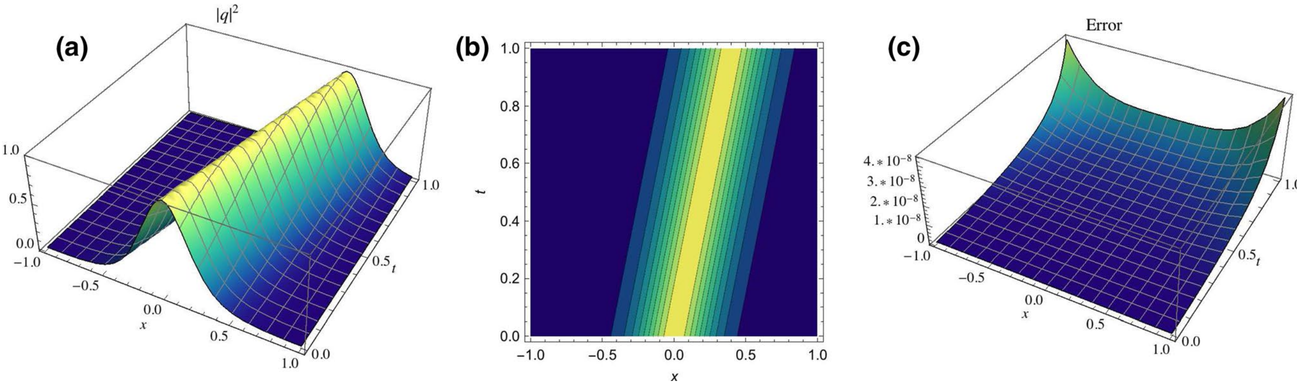

Fig. 1 A graphical representation of case 1: **a** a numerically computed bright soliton and **b** the corresponding contour plot, and **c** the absolute error in the form of a space–time graph for the 15th iteration of the LADM

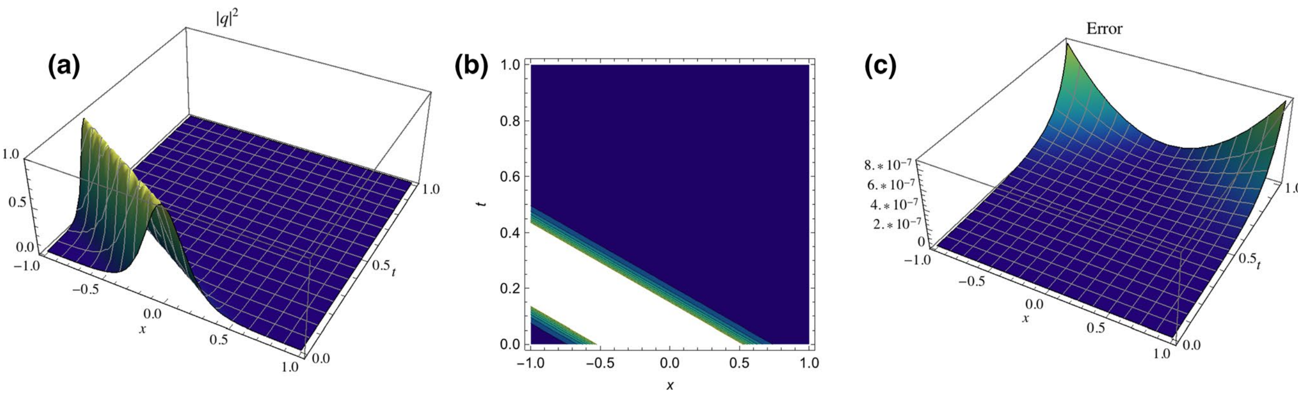

Fig. 2 A graphical representation of case 2: **a** a numerically computed bright soliton and **b** the corresponding contour plot, and **c** the absolute error in the form of a space–time graph for the 15th iteration of the LADM

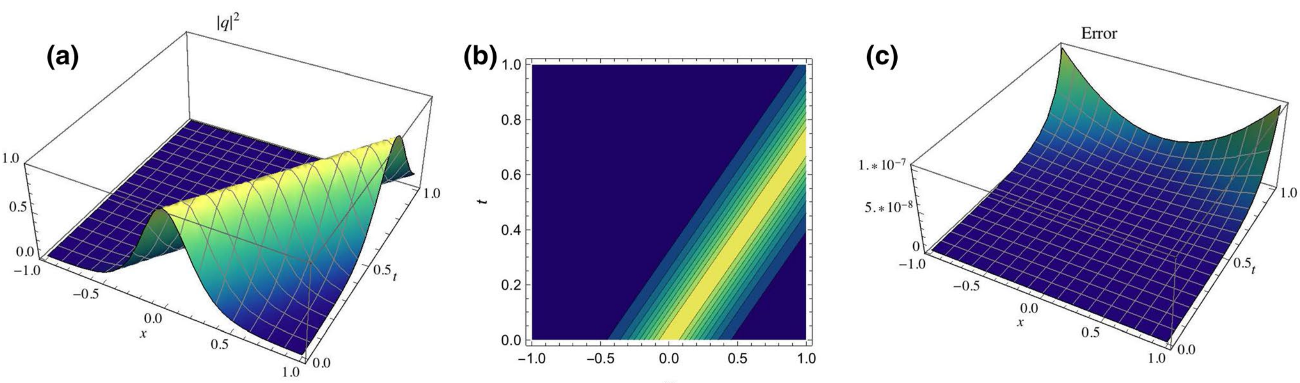

Fig. 3 A graphical representation of case 3: **a** a numerically computed bright soliton and **b** the corresponding contour plot, and **c** the absolute error in the form of a space–time graph for the 15th iteration of the LADM

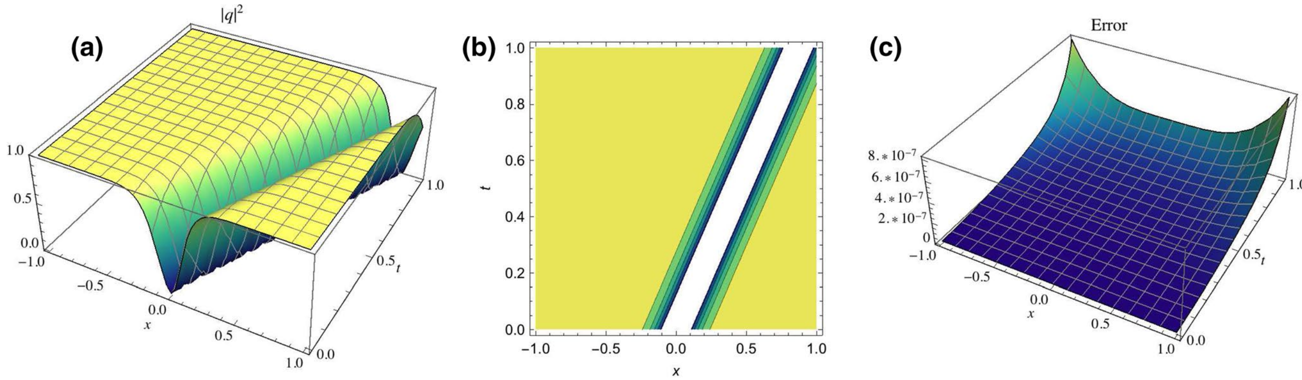

Fig. 4 A graphical representation of case 4: **a** a numerically computed dark soliton and **b** the corresponding contour plot, and **c** the absolute error in the form of a space-time graph for the 15th iteration of the LADM

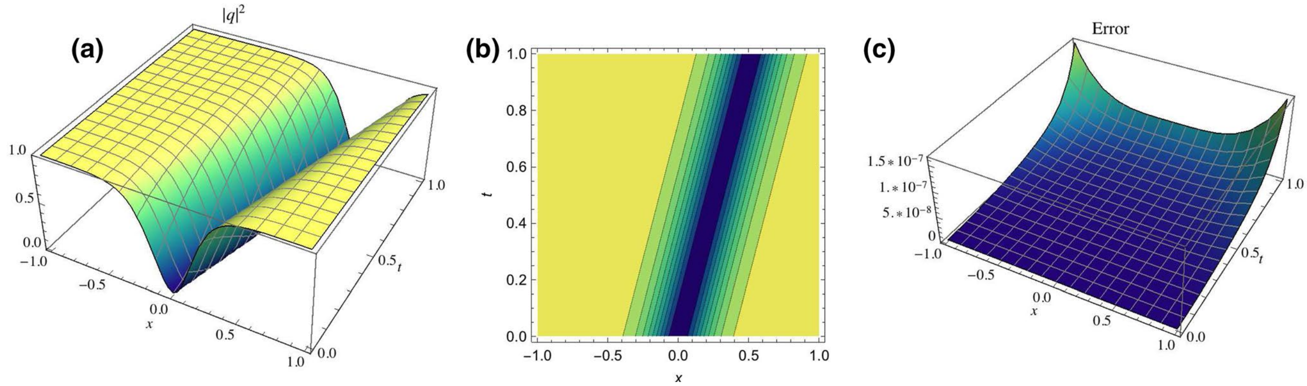

Fig. 5 A graphical representation of case 5: **a** a numerically computed dark soliton and **b** the corresponding contour plot, and **c** the absolute error in the form of a space–time graph for the 15th iteration of the LADM

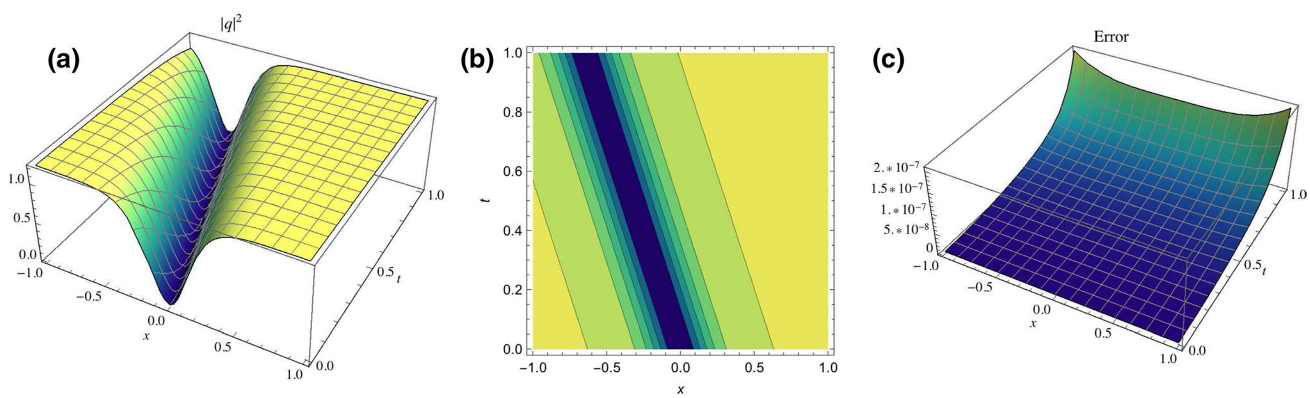

Fig. 6 A graphical representation of case 6: **a** a numerically computed dark soliton and **b** the corresponding contour plot, and **c** the absolute error in the form of a space–time graph for the 15th iteration of the LADM

4 Numerical implementation

In this section, we consider four different cases for the NLSE with CQS nonlinearity as given in Eq. (1) to illustrate the application of the LADM scheme presented above.

4.1 Bright highly dispersive optical solitons

We consider the initial condition at $t = 0$ from Eq. (2):

$$q(x, 0) = A\text{sech}(x) \times \exp\{i(-\kappa x + \Omega)\}. \tag{29}$$

We carry out the simulation for three cases with the parameters presented in Table 1; the results obtained as well as the absolute errors are shown in Figs. 1, 2, and 3.

4.2 Dark highly dispersive optical solitons

We now consider the initial condition at $t = 0$ from Eq. (5):

$$q(x, 0) = B\tanh(x) \times \exp\{i(-\kappa x + \Omega)\}. \tag{30}$$

We carry out the simulation for three cases with the parameters presented in Table 2; the results obtained as well as the absolute errors are shown in Figs. 4, 5, and 6.

5 Conclusions

Bright and dark optical solitons are illustrated from a numerical perspective. Sketches of the surface and contour plots of the bright and dark solitons are displayed. The error analysis of these numerical approximations is also tabulated and sketched. It is observed that the error values are on the order of $O(10^{-7})$, indicating that the numerical scheme is efficient and tolerant, and thus reliable.

Based on this groundwork, it is now time to move on to further developments. Our plan is to consider the use of these models of HD solitons, using the LADM approach, in birefringent fibers and later extend the numerical dynamics of such solitons with the dense wavelength division multiplexing (DWDM) topology. One of our future plans of study is the interaction of bright–dark solitons, which would lead to a whole new perspective on the study of HD solitons. Since bright and dark soliton solutions are available individually, it becomes necessary and imperative to carry out such investigations at least numerically. The results of such research will also be disseminated in the future. Such studies are underway and will soon be published.

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Declarations

Conflict of interest The authors declare they have no conflicts of interest.

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