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Optical solitons in fiber Bragg gratings with quadratic-cubic law of nonlinear refractive index and cubic-quartic dispersive reflectivity

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Abstract. This paper recovers cubic-quartic perturbed solitons in fiber Bragg gratings with quadratic-cubic law nonlinear refractive index. The unified Riccati equation expansion method and the modified Kudryashov's approach make this retrieval of soliton solutions possible. The parameter constraints, for the existence of such solitons, are also presented.

Keywords: solitons, cubic-quartic, Bragg gratings.

1. INTRODUCTION

The theory of solitons in fiber Bragg is one of the essential areas of research in nonlinear photonics and it has been studied both analytically and numerically [1–12]. The concept of fiber Bragg gratings (FBG) is a couple of decades old; however, the research on this topic remains active, but with an advanced and enhanced perspective. The concept of fiber Bragg gratings emerged when chromatic dispersion (CD) in soliton transmission dynamics got negligibly small and it is then that Bragg gratings were introduced in an optical fiber to compensate for the low-lying balance between CD and self-phase modulation (SPM). While this is a modern engineering marvel in the technology of optical fiber materials, another approach is to discard CD due to its low count and replace it collectively with third-order dispersive (3OD) and fourth-order dispersive (4OD) effects. This is known as cubic-quartic (CQ) dispersive effect. This paper will address optical fibers with fiber Bragg gratings with dispersive reflectivity from CQ dispersive effect

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[13–20]. The nonlinear law of refractive index will be sourced from quadratic-cubic (QC) terms. This is the model that will be analyzed in the paper using two integration algorithms which lead to a full spectrum of single soliton solutions that will be exhibited. The existence criteria of such solutions are also presented. The rest of the paper pens the derivation and displays these soliton solutions after a quick and succinct intro to the governing model.

1.1. Governing Model

The dimensionless form of the coupled cubic-quartic (CQ) perturbed nonlinear Schrödinger equation (NLSE) in fiber Bragg gratings (FBG) with quadratic-cubic law of nonlinear refractive index, is written as

$$\begin{aligned} &iq_t + ia_1 r_{xxx} + b_1 r_{xxxx} + c_1 q\sqrt{|q|^2+|r|^2+q^*r+qr^*} + \left(d_1|q|^2+e_1|r|^2\right)q \\ &+ i\alpha_1 q_x + \beta_1 r + \sigma_1 q^* r^2 = i\left[\gamma_1\left(|q|^2 q\right)_x + \theta_1\left(|q|^2\right)_x q + \mu_1 |q|^2 q_x\right], \end{aligned} \tag{1}$$

$$\begin{aligned} &ir_t + ia_2 q_{xxx} + b_2 q_{xxxx} + c_2 r\sqrt{|r|^2+|q|^2+r^*q+rq^*} + \left(d_2|r|^2+e_2|q|^2\right)r \\ &+ i\alpha_2 r_x + \beta_2 q + \sigma_2 r^* q^2 = i\left[\gamma_2\left(|r|^2 r\right)_x + \theta_2\left(|r|^2\right)_x r + \mu_2 |r|^2 r_x\right], \end{aligned} \tag{2}$$

where a_j and b_j $(j=1,2)$ are the coefficients of 3OD and 4OD, respectively. Here $q(x,t)$ and $r(x,t)$ represent forward and backward propagation wave profile, respectively, as long as $i=\sqrt{-1}$. Then, d_j are the coefficients of SPM terms; c_j and e_j represent the cross-phase modulation (XPM) terms; α_j represent the inter-modal dispersion and β_j are the detuning parameters. Also, σ_j give the four wave-mixing (4WM) terms, while γ_j represent the self-steepening terms to avoid the shock waves formulation. Finally, θ_j and μ_j read as the nonlinear dispersion coefficients.

The system (1) and (2) is a manifested version of the standard model, which comes with CD instead of CQ dispersive effect, and is structured as

$$\begin{aligned} &iq_t + a_1 r_{xx} + c_1 q\sqrt{|q|^2+|r|^2+q^*r+qr^*} + \left(d_1|q|^2+e_1|r|^2\right)q \\ &+ i\alpha_1 q_x + \beta_1 r + \sigma_1 q^* r^2 = i\left[\gamma_1\left(|q|^2 q\right)_x + \theta_1\left(|q|^2\right)_x q + \mu_1 |q|^2 q_x\right], \end{aligned} \tag{3}$$

$$\begin{aligned} &ir_t + a_2 q_{xx} + c_2 r\sqrt{|r|^2+|q|^2+r^*q+rq^*} + \left(d_2|r|^2+e_2|q|^2\right)r \\ &+ i\alpha_2 r_x + \beta_2 q + \sigma_2 r^* q^2 = i\left[\gamma_2\left(|r|^2 r\right)_x + \theta_2\left(|r|^2\right)_x r + \mu_2 |r|^2 r_x\right], \end{aligned} \tag{4}$$

where a_j $(j=1,2)$ are the coefficients of CD. In the system (1) and (2), it is this CD that is replaced by 3OD and 4OD which formulate the dispersion effects. Note that, the system (3) and (4) has been discussed in [18], when $\gamma_j=\theta_j=\mu_j=0$, for $j=1,2$. The objective of this paper is to apply two algorithms techniques mentioned in the abstract to construct the optical soliton solutions of the CQ-system (1) and (2).

This article is organized as follows: Section 2 introduces the mathematical preliminaries. In Sections 3 and 4, the unified Riccati equation expansion method and the modified Kudryashov's method are applied. In Section 5, conclusions are drawn.

2. MATHEMATICAL PRELIMINARIES

In order to solve Eqs (1) and (2), we assume the hypothesis

$$\begin{aligned} q(x,t) &= \phi_1(\xi)\exp\left[i\psi(x,t)\right], \\ r(x,t) &= \phi_2(\xi)\exp\left[i\psi(x,t)\right], \end{aligned} \tag{5}$$

such that

$$\xi = x - vt, \quad \psi(x,t) = -\kappa x + \omega t + \theta_0, \tag{6}$$

where v, κ, ω and θ_0 are all nonzero constants to be determined, which represent soliton velocity, soliton frequency, wave number and phase constant, respectively. Next, $\psi(x,t)$ is a real function which represents the phase component of the soliton, while $\phi_j(\xi)$ are real functions which represent the shape of the pulse of the solitons. Substituting (5) and (6) into Eqs (1) and (2), separating the real and imaginary parts, we deduce that the real parts are

$$\begin{aligned} &b_1\phi_2'''' + 3\left(a_1\kappa - 2b_1\kappa^2\right)\phi_2'' + \left(\beta_1 - a_1\kappa^3 + b_1\kappa^4\right)\phi_2 + \left(\alpha_1\kappa - \omega\right)\phi_1 \\ &+ c_1\phi_1\sqrt{\phi_1^2 + 2\phi_1\phi_2 + \phi_2^2} + \left[d_1 - \left(\mu_1 + \gamma_1\right)\kappa\right]\phi_1^3 + \left(\sigma_1 + e_1\right)\phi_1\phi_2^2 = 0, \end{aligned} \tag{7}$$

$$\begin{aligned} &b_2\phi_1'''' + 3\left(a_2\kappa - 2b_2\kappa^2\right)\phi_1'' + \left(\beta_2 - a_2\kappa^3 + b_2\kappa^4\right)\phi_1 + \left(\alpha_2\kappa - \omega\right)\phi_2 \\ &+ c_2\phi_2\sqrt{\phi_2^2 + 2\phi_2\phi_1 + \phi_1^2} + \left[d_2 - \left(\mu_2 + \gamma_2\right)\kappa\right]\phi_2^3 + \left(\sigma_2 + e_2\right)\phi_2\phi_1^2 = 0, \end{aligned} \tag{8}$$

while the imaginary parts are

$$\left(a_1 - 4b_1\kappa\right)\phi_2''' + \left(4b_1\kappa^3 - 3a_1\kappa^2\right)\phi_2' + (\alpha_1 - v)\phi_1' - \left(3\gamma_1 + 2\theta_1 + \mu_1\right)\phi_1^2\phi_1' = 0, \tag{9}$$

$$\left(a_2 - 4b_2\kappa\right)\phi_1''' + \left(4b_2\kappa^3 - 3a_2\kappa^2\right)\phi_1' + (\alpha_2 - v)\phi_2' - \left(3\gamma_2 + 2\theta_2 + \mu_2\right)\phi_2^2\phi_2' = 0. \tag{10}$$

Set

$$\phi_2(\xi) = A\phi_1(\xi), \tag{11}$$

where $A \neq 0$ or 1. Consequently, the real parts change into

$$\begin{aligned} &b_1A\phi_1'''' + 3\left(a_1\kappa - 2b_1\kappa^2\right)A\phi_1'' + \left[\left(\beta_1 - a_1\kappa^3 + b_1\kappa^4\right)A + \alpha_1\kappa - \omega\right]\phi_1 \\ &+ c_1\left(1 + A\right)\phi_1^2 + \left[d_1 - \left(\mu_1 + \gamma_1\right)\kappa + \left(\sigma_1 + e_1\right)A^2\right]\phi_1^3 = 0, \end{aligned} \tag{12}$$

$$\begin{aligned} &b_2\phi_1'''' + 3\left(a_2\kappa - 2b_2\kappa^2\right)\phi_1'' + \left[\beta_2 - a_2\kappa^3 + b_2\kappa^4 + (\alpha_2\kappa - \omega)A\right]\phi_1 \\ &+ c_2A\left(1 + A\right)\phi_1^2 + \left\{\left[d_2 - \left(\mu_2 + \gamma_2\right)\kappa\right]A^3 + \left(\sigma_2 + e_2\right)A\right\}\phi_1^3 = 0, \end{aligned} \tag{13}$$

while the imaginary parts become

$$\left(a_1 - 4b_1\kappa\right)A\phi_1''' + \left[\left(4b_1\kappa^3 - 3a_1\kappa^2\right)A + \alpha_1 - v\right]\phi_1' - \left(3\gamma_1 + 2\theta_1 + \mu_1\right)\phi_1^2\phi_1' = 0, \tag{14}$$

$$\left(a_2 - 4b_2\kappa\right)\phi_1''' + \left[4b_2\kappa^3 - 3a_2\kappa^2 + (\alpha_2 - v)A\right]\phi_1' - \left(3\gamma_2 + 2\theta_2 + \mu_2\right)A^3\phi_1^2\phi_1' = 0. \tag{15}$$

The linearly independent principle is applied on (14) and (15) to get the frequency of soliton as

$$\kappa = \frac{a_j}{4b_j}, \quad a_j \neq 0 \quad b_j \neq 0 \quad \text{for} \quad j = 1,2, \quad a_1 b_2 = a_2 b_1; \tag{16}$$

the velocity of soliton as

$$v = \left(4b_1\kappa^3 - 3a_1\kappa^2\right)A + \alpha_1 \tag{17}$$

or

$$v = \frac{4b_2\kappa^3 - 3a_2\kappa^2 + A\alpha_2}{A}; \tag{18}$$

and the constraint conditions:

$$3\gamma_j + 2\theta_j + \mu_j = 0 \quad \text{for} \quad j = 1,2. \tag{19}$$

From (17) and (18), one gets the following constraint condition:

$$4\left(b_1A^2 - b_2\right)\kappa^3 - 3\left(a_1A^2 - a_2\right)\kappa^2 + (\alpha_1 - \alpha_2)A = 0. \tag{20}$$

Eqs (12) and (13) have the same form under the constraint conditions:

$$\begin{aligned}
&b_1A = b_2,\\
&(a_1 - 2b_1\kappa)A = a_2 - 2b_2\kappa,\\
&\left(\beta_1 - a_1\kappa^3 + b_1\kappa^4\right)A + \alpha_1\kappa - \omega = \beta_2 - a_2\kappa^3 + b_2\kappa^4 + (\alpha_2\kappa - \omega)A,\\
&c_1 = c_2A,\\
&d_1 - (\mu_1 + \gamma_1)\kappa + (\sigma_1 + e_1)A^2 = [d_2 - (\mu_2 + \gamma_2)\kappa]A^3 + (\sigma_2 + e_2)A.
\end{aligned} \tag{21}$$

From (21), we have

$$\begin{aligned}
&\omega = \frac{\beta_1A - \beta_2 + (\alpha_1 - \alpha_2A)\kappa - (a_1A - a_2)\kappa^3}{1 - A},\\
&A = \frac{b_2}{b_1} = \frac{c_1}{c_2} = \frac{a_2 - 2b_2\kappa}{a_1 - 2b_1\kappa}, \quad b_1 \neq b_2, \quad c_1 \neq c_2 \quad \text{and} \quad a_1 \neq a_2.
\end{aligned} \tag{22}$$

Eq. (12) can be rewritten as

$$\phi_1'''' + \Delta_0\phi_1'' + \Delta_1\phi_1 + \Delta_2\phi_1^2 + \Delta_3\phi_1^3 = 0, \tag{23}$$

where

$$\begin{aligned}
&\Delta_0 = \frac{3\left(a_1\kappa - 2b_1\kappa^2\right)}{b_1}, \quad \Delta_1 = \frac{\left(\beta_1 - a_1\kappa^3 + b_1\kappa^4\right)A + \alpha_1\kappa - \omega}{b_1A},\\
&\Delta_2 = \frac{c_1(1 + A)}{b_1A}, \quad \Delta_3 = \frac{d_1 - (\mu_1 + \gamma_1)\kappa + (\sigma_1 + e_1)A^2}{b_1A}.
\end{aligned} \tag{24}$$

Next, Eq. (23) will be examined.

3. APPLICATION OF UNIFIED RICCATI EQUATION METHOD

According to this method, balancing ϕ_1'''' and ϕ_1^3 in Eq. (23), we get the balance number $N=2$. Thus, Eq. (23) has the formal solution

$$\phi_1(\xi) = \delta_0 + \delta_1 F(\xi) + \delta_2 F^2(\xi), \tag{25}$$

where δ_0, δ_1 and δ_2 are constants to be determined, provided $\delta_2 \neq 0$, while $F(\xi)$ satisfies the Riccati equation

$$F'(\xi) = h_0 + h_1 F(\xi) + h_2 F^2(\xi). \tag{26}$$

Here h_0, h_1 and h_2 are constants to be determined provided $h_2 \neq 0$. It is well known that the Riccati equation (26) has the following fractional solutions:

$$F(\xi) = -\frac{h_1}{2h_2} - \frac{\sqrt{\Delta}\left[r_1 \tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right) + r_2\right]}{2h_2\left[r_1 + r_2 \tanh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right]}, \quad \text{if} \quad \Delta > 0 \quad \text{and} \quad r_1^2 + r_2^2 \neq 0; \tag{27}$$

$$F(\xi) = -\frac{h_1}{2h_2} + \frac{\sqrt{-\Delta}\left[r_3 \tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right) - r_4\right]}{2h_2\left[r_3 + r_4 \tan\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right]}, \quad \text{if} \quad \Delta < 0 \quad \text{and} \quad r_3^2 + r_4^2 \neq 0; \tag{28}$$

$$F(\xi) = -\frac{h_1}{2h_2} + \frac{1}{h_2\xi + r_5}, \quad \text{if} \quad \Delta = 0; \tag{29}$$

where $\Delta = h_1^2 - 4h_0h_2$ and $r_i (i = 1, 2, ..., 5)$ are arbitrary constants. Substituting Eqs (25) and (26) into Eq. (23), collecting all the coefficients of $F^j(\xi)$ $(j = 0, 1, 2, ..., 6)$, and setting these coefficients to zero, one finds the following system of algebraic equations:

$$\begin{aligned}
&\Delta_3\delta_2^3 + 120h_2^4\delta_2 = 0,\\
&336h_2^3\delta_2h_1 + 3\Delta_3\delta_1\delta_2^2 + 24h_2^4\delta_1 = 0,\\
&6\Delta_0\delta_2h_2^2 + 3\Delta_3\delta_1^2\delta_2 + 240h_2^3\delta_2h_0 + \Delta_2\delta_2^2 + 60h_2^3\delta_1h_1 + 330h_2^2\delta_2h_1^2 + 3\Delta_3\delta_0\delta_2^2 = 0,\\
&14h_1^2h_0^2\delta_2 + h_1^3\delta_1h_0 + 2\Delta_0\delta_2h_0^2 + 16\delta_2h_0^3h_2 + \Delta_3\delta_0^3 + 8h_0^2h_2\delta_1h_1 + \Delta_1\delta_0 + \Delta_2\delta_0^2 + \Delta_0\delta_1h_1h_0 = 0,\\
&440h_2^2\delta_2h_1h_0 + 130h_1^3\delta_2h_2 + 2\Delta_0\delta_1h_2^2 + 40h_2^3\delta_1h_0 + 6\Delta_3\delta_0\delta_1\delta_2 + 50h_2^2\delta_1h_1^2\\
&\quad + 2\Delta_2\delta_1\delta_2 + \Delta_3\delta_1^3 + 10\Delta_0\delta_2h_1h_2 = 0,\\
&6\Delta_0\delta_2h_0h_1 + 22h_2\delta_1h_1^2h_0 + \Delta_1\delta_1 + 2\Delta_2\delta_0\delta_1 + 2\Delta_0\delta_1h_2h_0 + 3\Delta_3\delta_0^2\delta_1 + 16h_0^2h_2^2\delta_1\\
&\quad + 120h_2\delta_2h_0^2h_1 + 30h_1^3\delta_2h_0 + \Delta_0\delta_1h_1^2 + h_1^4\delta_1 = 0,\\
&136h_2^2\delta_2h_0^2 + 16h_1^4\delta_2 + 2\Delta_2\delta_0\delta_2 + 3\Delta_3\delta_0\delta_1^2 + 4\Delta_0\delta_2h_1^2 + \Delta_2\delta_1^2 + 3\Delta_0\delta_1h_1h_2 + 15h_2\delta_1h_1^3\\
&\quad + 8\Delta_0\delta_2h_0h_2 + 232h_1^2\delta_2h_2h_0 + \Delta_1\delta_2 + 60h_2^2\delta_1h_1h_0 + 3\Delta_3\delta_0^2\delta_2 = 0.
\end{aligned} \tag{30}$$

On solving the above algebraic Eq. (30) by using Maple, one gets the following results:

$$\delta_0 = \frac{\Delta_0\sqrt{-30\Delta_3} - 10\Delta_2}{10\Delta_3}, \quad \delta_1 = 0, \quad \delta_2 = \frac{2h_2^2\sqrt{-30\Delta_3}}{\Delta_3},$$
$$h_0 = \frac{\Delta_2\sqrt{-30\Delta_3} + 3\Delta_0\Delta_3}{60h_2\Delta_3}, \quad h_1 = 0, \quad h_2 = h_2, \tag{31}$$

and the constraint conditions:

$$\Delta_1 = \frac{12\Delta_0^2\Delta_3 + 10\Delta_2^2 + 3\Delta_0\Delta_2\sqrt{-30\Delta_3}}{75\Delta_3}, \tag{32}$$

provided $\Delta_3 < 0$. By the aid of solutions (27) and (28), we find the following types of solutions for Eqs (1) and (2).

Type 1. If $\Delta = h_1^2 - 4h_0h_2 > 0$, then substituting (31) along with (27) into Eq. (25), we have dark soliton solutions of Eqs (1) and (2) as

$$\begin{aligned} q(x,t) =& \frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\left\{-1 + \frac{\left[r_1 \tanh\left(\frac{1}{2}\sqrt{-\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right) + r_2\right]^2}{\left[r_1 + r_2\tanh\left(\frac{1}{2}\sqrt{-\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\right]^2}\right\} \\ & \times \exp\left[i(-\kappa x + \omega t + \theta_0\right], \end{aligned} \tag{33}$$

$$\begin{aligned} r(x,t) =& A\left(\frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\right)\left\{-1 + \frac{\left[r_1 \tanh\left(\frac{1}{2}\sqrt{-\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right) + r_2\right]^2}{\left[r_1 + r_2\tanh\left(\frac{1}{2}\sqrt{-\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\right]^2}\right\} \\ & \times \exp\left[i(-\kappa x + \omega t + \theta_0\right], \end{aligned} \tag{34}$$

provided $\Delta_2\sqrt{-30\Delta_3} + 3\Delta_0\Delta_3 > 0$ and $\Delta_3 < 0$.
In particular if $r_1 \neq 0$ and $r_2 = 0$, then one gets the bright soliton solutions of Eqs (1) and (2) in the form

$$q(x,t) = -\frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\operatorname{sech}^2\left(\frac{1}{2}\sqrt{-\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\exp\left[i(-\kappa x + \omega t + \theta_0\right], \tag{35}$$

$$r(x,t) = A\left(-\frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\right)\operatorname{sech}^2\left(\frac{1}{2}\sqrt{-\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\exp\left[i(-\kappa x + \omega t + \theta_0\right], \tag{36}$$

while, if $r_1 = 0$ and $r_2 \neq 0$, then we have the singular soliton solutions of Eqs (1) and (2) in the form

$$q(x,t) = \frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\operatorname{csch}^2\left(\frac{1}{2}\sqrt{-\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\exp\left[i(-\kappa x + \omega t + \theta_0\right], \tag{37}$$

$$r(x,t) = A\left(\frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\right)\operatorname{csch}^2\left(\frac{1}{2}\sqrt{-\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\exp\left[i(-\kappa x+\omega t+\theta_0\right]. \tag{38}$$

The solutions (33)–(38) exist under the constraint conditions (32).

Type 2. If $\Delta = h_1^2 - 4h_0h_2 < 0$, then substituting (31) along with (28) into Eq. (25), we obtain the periodic wave solutions of Eqs (1) and (2) in the form

$$\begin{aligned} q(x,t) = &-\frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\left\{1+\frac{\left[r_3\tan\left(\frac{1}{2}\sqrt{\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)-r_4\right]^2}{\left[r_3+r_4\tan\left(\left(\frac{1}{2}\sqrt{\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\right)\right]^2}\right\} \\ &\times\exp\left[i(-\kappa x+\omega t+\theta_0\right], \end{aligned} \tag{39}$$

$$\begin{aligned} r(x,t) = &A\left(-\frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\right)\left\{1+\frac{\left[r_3\tan\left(\frac{1}{2}\sqrt{\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)-r_4\right]^2}{\left[r_3+r_4\tan\left(\left(\frac{1}{2}\sqrt{\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\right)\right]^2}\right\} \\ &\times\exp\left[i(-\kappa x+\omega t+\theta_0\right], \end{aligned} \tag{40}$$

provided $\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3 < 0$ and $\Delta_3 < 0$.
In particular if $r_3 \neq 0$ and $r_4 = 0$, then Eqs (1) and (2) have the periodic wave solutions in the form

$$q(x,t) = -\frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\sec^2\left(\frac{1}{2}\sqrt{\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\exp\left[i(-\kappa x+\omega t+\theta_0\right], \tag{41}$$

$$r(x,t) = A\left(-\frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\right)\sec^2\left(\frac{1}{2}\sqrt{\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\exp\left[i(-\kappa x+\omega t+\theta_0\right], \tag{42}$$

while, if $r_3 = 0$ and $r_4 \neq 0$, then Eqs (1) and (2) have the periodic wave solutions in the form

$$q(x,t) = -\frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\csc^2\left(\frac{1}{2}\sqrt{\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\exp\left[i(-\kappa x+\omega t+\theta_0\right], \tag{43}$$

$$r(x,t) = A\left(-\frac{10\Delta_2 - \Delta_0\sqrt{-30\Delta_3}}{10\Delta_3}\right)\csc^2\left(\frac{1}{2}\sqrt{\frac{\Delta_2\sqrt{-30\Delta_3}+3\Delta_0\Delta_3}{15\Delta_3}}\,(x-vt)\right)\exp\left[i(-\kappa x+\omega t+\theta_0\right]. \tag{44}$$

The solutions (39)–(44) exist under the constraint conditions (32).

4. APPLICATION OF MODIFIED KUDRYASHOV'S METHOD

There are several approaches and results of Kudryashov that have gained a considerable popularity lately due to its unique integrability structure [21–30]. Modified Kudryashov's approach proposes the formal solution to Eq. (23) as given below

$$\phi_1(\xi) = \sum_{j=0}^{N} \rho_j \left[Q(\xi)\right]^j, \tag{45}$$

where ρ_j are constants to be determined later, provided $\rho_N \neq 0$ and $Q(\xi)$ takes the form

$$Q'(\xi) = Q(\xi)\left[Q^p(\xi) - 1\right]\ln a, \quad 0 < a \neq 1, \tag{46}$$

with the positive integer p, such that the solutions of Eq. (46) are given by

$$Q(\xi) = \left[\frac{1}{1 + \varepsilon \exp_a(p\xi + \xi_0)}\right]^{\frac{1}{p}}, \tag{47}$$

where $\exp_a(p\xi + \xi_0) = a^{p\xi + \xi_0}$, ξ_0 is a constant and $\varepsilon = \pm 1$. Eq. (47) can be rewritten in combination with bright-singular soliton solution as

$$Q(\xi) = \left[\frac{1}{1 + \varepsilon\left\{\cosh\left[(p\xi + \xi_0)\ln a\right] + \sinh\left[(p\xi + \xi_0)\ln a\right]\right\}}\right]^{\frac{1}{p}}. \tag{48}$$

In particular if $\varepsilon = 1$, one gets the dark soliton solution

$$Q(\xi) = \left[\frac{1}{2}\left\{1 - \tanh\left[\frac{(p\xi + \xi_0)}{2}\ln a\right]\right\}\right]^{\frac{1}{p}}. \tag{49}$$

When $\varepsilon = -1$, the singular soliton solution is obtained

$$Q(\xi) = \left[\frac{1}{2}\left\{1 - \coth\left[\frac{(p\xi + \xi_0)}{2}\ln a\right]\right\}\right]^{\frac{1}{p}}. \tag{50}$$

Balancing ϕ_1'''' with ϕ_1^3 in Eq. (23), one gets the relation

$$N + 4p = 3N \Longrightarrow N = 2p. \tag{51}$$

Next, two cases will be studied.

<u>Case 1.</u> Let us choose $p = 1$, then $N = 2$. This means that the solution of Eq. (23) becomes

$$\phi_1(\xi) = \rho_0 + \rho_1 Q(\xi) + \rho_2 Q^2(\xi), \tag{52}$$

where ρ_0, ρ_1 and ρ_2 are constants to be determined, $\rho_2 \neq 0$ while $Q(\xi)$ holds

$$Q'(\xi) = Q(\xi)\left[Q(\xi) - 1\right]\ln a, \quad 0 < a \neq 1. \tag{53}$$

Substituting (52) and (53) into Eq. (23) and collecting all the coefficients of each power of $[Q(\xi)]^{m_1}$ ($m_1 = 0, 1, ..., 6$) and setting each of these coefficients to zero, one gets the following system of algebraic equations:

$$\begin{aligned}
&120\left(\ln^4 a\right)\rho_2 + \Delta_3\rho_2^3 = 0,\\
&\Delta_1\rho_0 + \Delta_2\rho_0^2 + \Delta_3\rho_0^3 = 0,\\
&[24\rho_1 - 336\rho_2]\left(\ln^4 a\right) + 3\Delta_3\rho_1\rho_2^2 = 0,\\
&[\rho_1 + \Delta_0\rho_1]\left(\ln^2 a\right) + 3\Delta_3\rho_0^2\rho_1 + 2\Delta_2\rho_0\rho_1 + \Delta_1\rho_1 = 0,\\
&330\left(\ln^4 a\right)\rho_2 + 6\Delta_0\left(\ln^2 a\right)\rho_2 + 3\Delta_3\rho_1^2\rho_2 + \Delta_2\rho_2^2 + 3\Delta_3\rho_0\rho_2^2 - 60\left(\ln^4 a\right)\rho_1 = 0,\\
&6\Delta_3\rho_0\rho_1\rho_2 + [50\rho_1 - 130\rho_2]\left(\ln^4 a\right) + [2\Delta_0\rho_1 - 10\Delta_0\rho_2]\left(\ln^2 a\right) + 2\Delta_2\rho_1\rho_2 + \Delta_3\rho_1^3 = 0,\\
&\Delta_1\rho_2 + 3\Delta_3\rho_0^2\rho_2 + 3\Delta_3\rho_0\rho_1^2 + \Delta_2\rho_1^2 + [4\Delta_0\rho_2 - 3\Delta_0\rho_1]\left(\ln^2 a\right)\\
&\quad + [16\rho_2 - 15\rho_1]\left(\ln^4 a\right) + 2\Delta_2\rho_0\rho_2 = 0.
\end{aligned} \tag{54}$$

Solving the resulting system (54) by using Maple yields

$$\rho_0 = 0, \quad \rho_1 = \frac{2\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3}, \quad \rho_2 = -\frac{2\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3}, \tag{55}$$

and the constraint conditions

$$\Delta_0 = \frac{\Delta_2\sqrt{-30\Delta_3} - 15\Delta_3\ln^2 a}{3\Delta_3}, \quad \Delta_1 = \frac{\left(12\Delta_3\ln^2 a - \Delta_2\sqrt{-30\Delta_3}\right)\ln^2 a}{3\Delta_3}, \tag{56}$$

provided $\Delta_3 < 0$. Substituting (55) along with (48) into Eq. (52), one gets the combo bright-singular soliton solutions of Eqs (1) and (2) as

$$\begin{aligned}
q(x,t) =& \frac{2\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3}\left[\frac{1}{1+\varepsilon\left\{\cosh[(x-vt)\ln a] + \sinh[(x-vt)\ln a]\right\}}\right.\\
&\left. - \frac{1}{\left(1+\varepsilon\left\{\cosh[(x-vt)\ln a] + \sinh[(x-vt)\ln a]\right\}\right)^2}\right]\exp[i(-\kappa x + \omega t + \theta_0],
\end{aligned} \tag{57}$$

$$\begin{aligned}
r(x,t) =& \frac{2A\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3}\left[\frac{1}{1+\varepsilon\left\{\cosh[(x-vt)\ln a] + \sinh[(x-vt)\ln a]\right\}}\right.\\
&\left. - \frac{1}{\left(1+\varepsilon\left\{\cosh[(x-vt)\ln a] + \sinh[(x-vt)\ln a]\right\}\right)^2}\right]\exp[i(-\kappa x + \omega t + \theta_0].
\end{aligned} \tag{58}$$

In particular if $\varepsilon = 1$, bright soliton emerges as

$$q(x,t) = \frac{\sqrt{-30\Delta_3}\ln^2 a}{2\Delta_3}\operatorname{sech}^2\left[\frac{\ln a}{2}(x-vt)\right]\exp[i(-\kappa x + \omega t + \theta_0]. \tag{59}$$

$$r(x,t)=\frac{A\sqrt{-30\Delta_3}\ln^2 a}{2\Delta_3}\operatorname{sech}^2\left[\frac{\ln a}{2}(x-vt)\right]\exp[i(-\kappa x+\omega t+\theta_0],\tag{60}$$

When $\varepsilon=-1$, the following singular soliton is procured:

$$q(x,t)=-\frac{\sqrt{-30\Delta_3}\ln^2 a}{2\Delta_3}\operatorname{csch}^2\left[\frac{\ln a}{2}(x-vt)\right]\exp[i(-\kappa x+\omega t+\theta_0],\tag{61}$$

$$r(x,t)=-\frac{A\sqrt{-30\Delta_3}\ln^2 a}{2\Delta_3}\operatorname{csch}^2\left[\frac{\ln a}{2}(x-vt)\right]\exp[i(-\kappa x+\omega t+\theta_0].\tag{62}$$

The solutions (57)–(62) exist under the constraint conditions (56).

Case 2. Choosing $p=2$, then $N=4$. Thus, the formal solution of Eq. (23) is structured as

$$\phi_1(\xi)=\rho_0+\rho_1 Q(\xi)+\rho_2 Q^2(\xi)+\rho_3 Q^3(\xi)+\rho_4 Q^4(\xi),\tag{63}$$

where $\rho_0,\rho_1,\rho_2,\rho_3$ and ρ_4 are constants to be established, such that $\rho_4\neq 0$ while $Q(\xi)$ satisfies the equation

$$Q'(\xi)=Q(\xi)\left[Q^2(\xi)-1\right]\ln a,\quad 0<a\neq 1.\tag{64}$$

Substituting (63) with (48) into (23), collecting all the coefficients of $[Q(\xi)]^{m_2}$ $(m_2=0,1,...,12)$, and setting these coefficients to zero, one finds a set of algebraic equations, which can be solved by Maple to get the following results:

$$\rho_0=\rho_1=\rho_3=0,\quad \rho_2=\frac{8\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3},\quad \rho_4=-\frac{8\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3},\tag{65}$$

and the constraint conditions:

$$\Delta_0=\frac{\Delta_2\sqrt{-30\Delta_3}-60\Delta_3\ln^2 a}{3\Delta_3},\quad \Delta_1=\frac{4\left(48\Delta_3\ln^2 a-\Delta_2\sqrt{-30\Delta_3}\right)\ln^2 a}{3\Delta_3},\tag{66}$$

provided $\Delta_3<0$. Inserting (65) with (64) into Eq. (63), one obtains the combo bright-singular soliton solutions of Eqs (1) and (2) as

$$\begin{aligned}q(x,t)=&\frac{8\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3}\left[\frac{1}{1+\varepsilon\{\cosh[2(x-vt)\ln a]+\sinh[2(x-vt)\ln a]\}}\right.\\&\left.-\frac{1}{(1+\varepsilon\{\cosh[2(x-vt)\ln a]+\sinh[2(x-vt)\ln a]\})^2}\right]\exp[i(-\kappa x+\omega t+\theta_0],\end{aligned}\tag{67}$$

$$\begin{aligned}r(x,t)=&\frac{8A\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3}\left[\frac{1}{1+\varepsilon\{\cosh[2(x-vt)\ln a]+\sinh[2(x-vt)\ln a]\}}\right.\\&\left.-\frac{1}{(1+\varepsilon\{\cosh[2(x-vt)\ln a]+\sinh[2(x-vt)\ln a]\})^2}\right]\exp[i(-\kappa x+\omega t+\theta_0].\end{aligned}\tag{68}$$

In particular if $\varepsilon=1$, bright soliton emerges as

$$q(x,t)=\frac{2\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3}\operatorname{sech}^2[(x-vt)\ln a]\exp[i(-\kappa x+\omega t+\theta_0],\tag{69}$$

$$r(x,t) = \frac{2A\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3}\operatorname{sech}^2[(x-vt)\ln a]\exp[i(-\kappa x+\omega t+\theta_0)]. \tag{70}$$

When $\varepsilon = -1$, the following singular soliton is procured:

$$q(x,t) = -\frac{2\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3}\operatorname{csch}^2[(x-vt)\ln a]\exp[i(-\kappa x+\omega t+\theta_0)], \tag{71}$$

$$r(x,t) = -\frac{2A\sqrt{-30\Delta_3}\ln^2 a}{\Delta_3}\operatorname{csch}^2[(x-vt)\ln a]\exp[i(-\kappa x+\omega t+\theta_0)]. \tag{72}$$

The solutions (67)–(72) exist under the constraint conditions (66).

Similarly, one can derive many other solutions by selecting other values for p and N.

5. CONCLUSIONS

This paper recovered single optical soliton solutions in fiber Bragg gratings that come with CQ dispersive reflectivity with QC nonlinear law of refractive index. The model includes a few Hamiltonian type perturbation terms. Two integration algorithms have made this retrieval possible. Single soliton solutions of all kinds are recovered and are listed with their respective parameter constraints. The results of this form of a model are being reported for the first time and a lot of work lies ahead of us with such a display. One of the immediate topics to address is the conservation laws that would make a big difference as this essential feature is noticeably missing in the literature of Bragg gratings. Therefore, it is imperative to bridge this gap. Many other features such as addressing this model with fractional temporal evolution or time-dependent coefficients or even handling the study of solitons, with Bragg gratings, using Lie symmetry, are all yet to be done along the lines of the previously reported studies [31–40]. These ambitious projects still form only the tip of the iceberg! One must also make a cautious approach and try to avoid reporting solutions that will later be identified as false [31].

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