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Analytic study on interactions between periodic solitons with controllable parameters

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Abstract Soliton interactions occur when two solitons are close enough. In general, periodic oscillations can be presented during soliton interactions. The periodic oscillations will lead to the soliton distortion, which is necessary to carry out the effective control. In this paper, interactions between periodic solitons with controllable parameters are investigated analytically. One- and two-soliton solutions for the nonlinear Schrödinger equation are derived by using the Hirota's bilinear method. According to analytic solutions, the influences of each parameter on period interactions between solitons are discussed, and the method of how to control the cycle of interactions is suggested. Results in this paper can be used for the theoretical guidance of how to make the soliton transmission more efficient and more fidelity, and are of great significance for optical fiber communications.

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1 Introduction

In mathematical physics, solitons are a class of solutions to nonlinear evolution equations that describe weakly nonlinear dispersions of various physical systems [1–18]. Since the soliton was discovered, the research on optical solitons has always been very active, and great progress has been made both theoretically and experimentally [19–29]. Solitons have such advantages as constant waveform, constant speed, good confidentiality and high fidelity; therefore, they are used in the field of optical communications [30]. Both the fidelity and speed of the soliton transmission are indicators of the enhanced quality of communication systems. Higher quality and faster speed of information

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transmission are needed. As we all know, the nonlinear Schrödinger equation (NLSE)

$$i\frac{\partial u}{\partial t}+2|q|^2q+\frac{\partial q}{\partial x^2}=0 \tag{1}$$

can be used to describe the transmission of picosecond optical pulses in optical fibers [31–35]. In nonlinear optics, solitons are divided into two categories: temporal solitons and spatial solitons. They are derived from the optical Kerr effect which is a phenomenon that the refractive index of the material alters caused by the change of light intensity nonlinearly, and such phenomenon results in the self-focusing or self-defocusing of optical pulses in the space.

However, for the transmission of sub-picosecond and femtosecond pulses which has a better performance on the transmitting information, higher-order effects should be taken into consideration. The higher-order integrable NLS hierarchy can be presented as [36–40]:

$$\begin{aligned}&iq_x+\alpha_2(q_{tt}+2q|q|^2)-i\alpha_3(q_{ttt}+6q_t|q|^2)\\&\quad+\alpha_4(q_{tttt}+6q^*q_t^2+4q|q_t|^2\\&\quad+8|q|^2q_{tt+2q^2q_{tt}^*+6|q|^4q})-i\alpha_5(q_{ttttt}\quad+10|q|^2q_{ttt}\\&\quad+30|q|^4q_t+10qq_tq_{tt}^*+10qq_t^*q_{tt}+10q_t^2q_t^*\\&\quad+20q^*q_tq_{tt})+\cdots=0,\end{aligned} \tag{2}$$

where $q(x,t)$ denotes the normalized complex amplitude of the optical pulse envelope and $*$ represents the conjugation. $\alpha_l(l=2,3,4\cdots)$ are real constant parameters. x and t are the propagation variable and transverse variable, respectively. The breather solutions of first-order ($\alpha_m=0, m=2,3,4\cdots$) and second-order ($\alpha_m=0, m=3,4\cdots$) NLS equations have been solved by the Darboux transformation (DT) [36]. However, to our knowledge, the analytic solution of third-order NLS equations has not been solved. In this paper, one- and two-soliton solutions of third-order NLS equations ($\alpha_m=0, m=4,5\cdots$) are given as

$$iq_x+\alpha_2(q_{tt}+2q|q|^2)-i\alpha_3(q_{ttt}+6q_t|q|^2)=0. \tag{3}$$

Among the variety of methods of finding analytic soliton solutions, Hirota's bilinear method is an important and direct method which is put forth creatively. This method takes the bilinear derivative as a tool and is only related to the equation to be solved. Besides, it does not depend on the spectral problem of the equation or the Lax pair. Thus, the Hirota's bilinear method is a concise, intuitive and distinctive method.

As a nonlinear phenomenon, the interaction between optical solitons is a natural phenomenon. When multiple optical solitons are transmitted through optical fibers, it is clear that they need to be separated from each other in order to ensure the integrity and accuracy of the information [41]. So each optical soliton is required to enter the fiber at a smaller interval, but it will cause interaction between adjacent optical solitons and decrease the bandwidth of optical communication systems. Those cases will weaken the communication capability of optical soliton transmission systems [42,43]. Therefore, studying the interaction between optical solitons based on the NLSE is the key to improve the quality and capacity of long-distance communication systems. Based on Eq. (3), interactions between periodic solitons with controllable parameters will be studied in this paper.

The structure of this paper is indicated as below. In Sect. 2, analytic one- and two-soliton solutions for Eq. (3) will be solved by Hirota's bilinear method. In Sect. 3, the influences of various parameters on soliton interactions and their period are discussed. Finally, in Sect. 4, our conclusions are given.

2 Analytic one- and two-soliton solutions for Eq. (3)

Analytic solutions for Eq. (3) are obtained by the Hirota's bilinear method through the introduction of dependent variable transformation [44,45]

$$q=\frac{h(x,t)}{f(x,t)}, \tag{4}$$

where $h(x,t)$ is a complex differentiable function and $f(x,t)$ is a real one. With bilinear operators, bilinear forms are obtained after symbolic manipulations:

$$(iD_x+\alpha_2D_t^2-\alpha_3D_t^3)h\cdot f=0, \tag{5}$$
$$D_t^2f\cdot f-2|h|^2=0. \tag{6}$$

The Hirota's bilinear operators D_x^n and D_x^n are defined by [46,47]

$$\begin{aligned}&D_x^lD_t^nh(x,t)\cdot f(x,t)\\&\quad=\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^l\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^nh(x,t)f(x',t')\bigg|_{x'=x,t'=t}.\end{aligned} \tag{7}$$

Equation (3) can be solved by the following formal parameter power series expansion for $h(x,t)$ and $f(x,t)$:

$$h(x,t) = \varepsilon h_1(x,t) + \varepsilon^3 h_3(x,t) + \varepsilon^5 h_5(x,t) + \cdots, \tag{8}$$

$$f(x,t) = 1 + \varepsilon^2 f_2(x,t) + \varepsilon^4 f_4(x,t) + \varepsilon^6 f_6(x,t) + \cdots, \tag{9}$$

where ε is a formal expansion parameters. Bringing the expansion Eqs. (8) and (9) into the bilinear equations Eqs. (5) and (6), making the coefficient of the same powers of ε equal to zero, can get the recursion relation of $h_n(x,t)$ and $f_n(x,t)$ $(n = 1, 2\cdots)$. To derive one-soliton solution for Eq. (3), assume that

$$h_1(x,t) = e^{\theta}, \tag{10}$$

$$h(x,t) = \varepsilon h_1(x,t), \tag{11}$$

$$f(x,t) = 1 + \varepsilon^2 f_2(x,t), \tag{12}$$

where $\theta = \psi(x) + \eta t + \phi_0$ with η as an arbitrary complex parameter, ψ_0 is a real constant, and $\psi(x)$ is an differentiable indeterminate function. Through substituting $h_1 = e^{\theta}$ into the linear partial differential equations consisting of recursion relations of $g_n(x,t)$ and $f_n(x,t)$, the calculation yields:

$$\psi(x) = (i\alpha_2\eta^2 + \alpha_3\eta^3)x, \tag{13}$$

$$f_2(x,t) = \frac{e^{\theta+\theta^*}}{(\eta+\eta^*)^2}, \tag{14}$$

$$h_n(x,t) = 0, (n = 3, 5\cdots), \tag{15}$$

$$f_n(x,t) = 0, (n = 4, 6\cdots). \tag{16}$$

Without loss of generality, take $\varepsilon = 1$. The expression of one-soliton solution is:

$$q(x,t) = \frac{h(x,t)}{f(x,t)} = \frac{e^{\theta}}{1 + \frac{e^{\theta+\theta^*}}{(\eta+\eta^*)^2}}. \tag{17}$$

To obtain the two-soliton solution for Eq. (3), assume that $h_1(x,t)$, $h(x,t)$ and $f(x,t)$ are as follows:

$$h_1(x,t) = e^{\theta_1} + e^{\theta_2}, \tag{18}$$

$$h(x,t) = \varepsilon h_1(x,t) + \varepsilon^3 h_3(x,t), \tag{19}$$

$$f(x,t) = 1 + \varepsilon^2 f_2(x,t) + \varepsilon^4 f_4(x,t), \tag{20}$$

where $\theta_j = \psi_j(x) + \eta_j t + \phi_j (j = 1, 2)$, η is an arbitrary complex parameter, $\psi_j(x) = (i\alpha_2\eta_j^2 + \alpha_3\eta_j^3)x$, and ϕ_j is a real constant. Substituting Eqs. (18)–(20) into Eqs. (5) and (6), and setting $\varepsilon = 1$, two-soliton solutions of Eq. (3) can be explicitly expressed as:

$$q(x,t) = \frac{h_1(x,t) + h_3(x,t)}{1 + f_2(x,t) + f_4(x,t)}, \tag{21}$$

where $h_1(x,t)$, $h_3(x,t)$, $f_2(x,t)$ and $f_4(x,t)$ are indicated as below:

$$h_1(x,t) = e^{\theta_1} + e^{\theta_2},$$
$$h_3(x,t) = A_1 e^{\theta_1+\theta_2+\theta_1^*} + A_2 e^{\theta_1+\theta_2+\theta_2^*},$$
$$f_2(x,t) = B_1 e^{\theta_1+\theta_1^*} + B_2 e^{\theta_2+\theta_1^*} + B_3 e^{\theta_1+\theta_2^*} + B_4 e^{\theta_2+\theta_2^*},$$
$$f_4(x,t) = C_1 e^{\theta_1+\theta_2+\theta_1^*+\theta_2^*},$$
$$A_1 = \frac{(\eta_2-\eta_1)^2}{(\eta_1^*+\eta_1)^2(\eta_1^*+\eta_2)^2},$$
$$A_2 = \frac{(\eta_2-\eta_1)^2}{(\eta_2^*+\eta_1)^2(\eta_2^*+\eta_2)^2},$$
$$B_1 = \frac{1}{(\eta_1^*+\eta_1)^2}, \quad B_2 = \frac{1}{(\eta_1^*+\eta_2)^2},$$
$$B_3 = \frac{1}{(\eta_2^*+\eta_1)^2}, \quad B_4 = \frac{1}{(\eta_2^*+\eta_2)^2},$$
$$C_1 = \frac{(\eta_1-\eta_2)^2(\eta_1^*-\eta_2^*)^2}{(\eta_1^*+\eta_1)^2(\eta_1^*+\eta_2)^2(\eta_2^*+\eta_1)^2(\eta_2^*+\eta_2)^2}.$$

3 Discussion

3.1 One-soliton solution parameter analysis

In the analytic one-soliton solution of Eq. (3), there are four parameters η, ϕ, α_2 and α_3. α_2 and α_3 are relative to second- and third-order dispersion and nonlinear coefficient which is the parameter to determine the basic nature of optical pulse propagation. Others parameters η and ϕ are variable parameters that have no connection with the basic nature of optical solitons. Firstly, in order to discuss the impact of different ϕ_0 on the optical solitons, we fix other values unchanged and choose the different values of ϕ_0. According to solution (17), we can get the graphics shown in Fig. 1.

As we can see, from Fig. 1a, b, $\phi_0 = 0$, two rows of solitons with the same shape but different peak positions are obtained. And the larger the value of ϕ_0 is, the more the soliton moves to the right. Obviously, ϕ_0 does not affect the pulse waveform and propagation direction. Not only that, the peak value will not change if ϕ_0 is taken different values. η is a complex number, and we might as well set $\eta = p_1 + p_2 i$ for discussing the real and imaginary parts separately, where p_1 and p_2 are both arbitrary real constants, so that

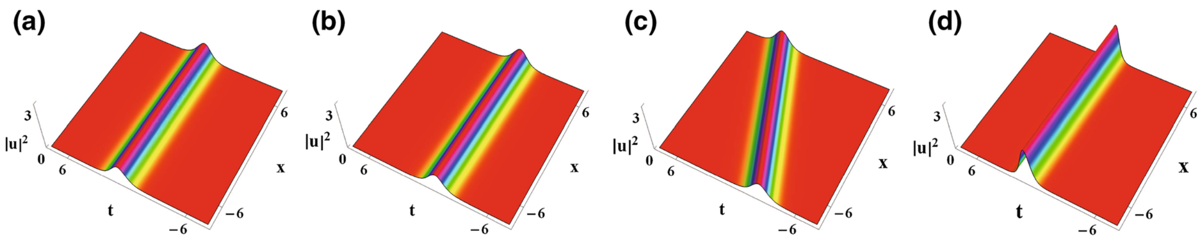

Fig. 1 One-soliton propagation for Eq. (3) with **a** $\eta = 1, \alpha_2 = 1, \alpha_3 = 0.01, \phi_0 = 0$. **b** $\eta = 1, \alpha_2 = 1, \alpha_3 = 0.01, \phi_0 = ln5$. **c** $\eta = 1 + 0.2i, \alpha_2 = 1, \alpha_3 = 0.01, \phi_0 = 0$. **d** $\eta = 1.5, \alpha_2 = 1, \alpha_3 = 0.01, \phi_0 = 0$

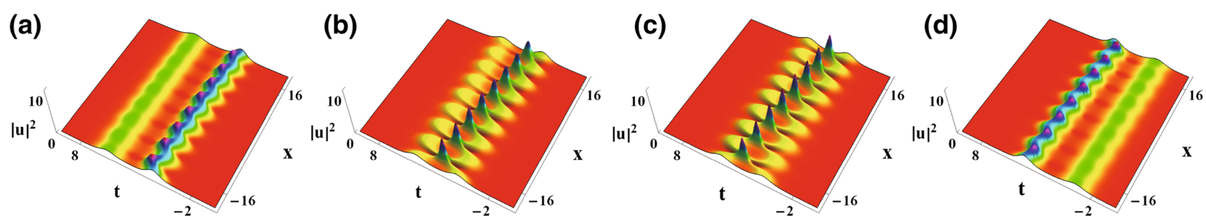

Fig. 2 Soliton interactions described by two-soliton solutions (21) with $\eta_1 = 1, \eta_2 = 1.5, \alpha_2 = 1, \alpha_3 = 0.01, \phi_2 = ln0.5$, **a** $\phi_1 = ln(0.25)$. **b** $\phi_1 = ln(0.75)$. **c** $\phi_1 = 0$. **d** $\phi_1 = ln(4)$

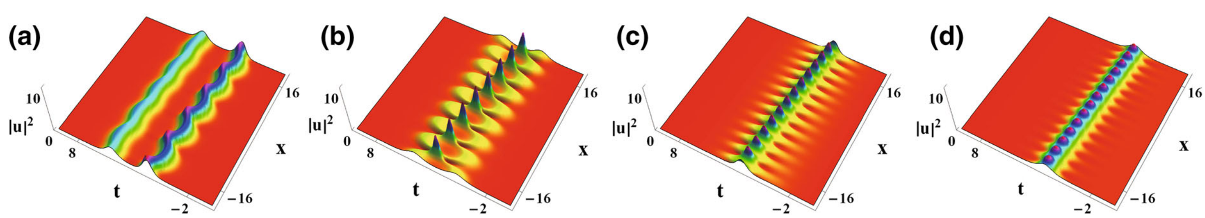

Fig. 3 Soliton interactions described by two-soliton solutions (21) with $\eta_2 = 1.5, \alpha_2 = 1, \alpha_3 = 0.01, \phi_2 = ln(0.5), \phi_1 = 0$. **a** $\eta_1 = 1.75$. **b** $\eta_1 = 1$. **c** $\eta_1 = 0.5$. **d** $\eta_1 = 0.25$

$\psi(x) = (i\alpha_2\eta^2 + \alpha_3\eta^3)x = [(-2p_1p_2\alpha_2 + 2p_1^3\alpha_3 - 3p_1p_2^2\alpha_3) + i((p_1^2 - p_2^2)\alpha_2 + (3p_1^2 - p_2^2)p_2\alpha_3)]x$. Keeping the same value of ϕ_0, when the imaginary part appears in η, the characteristics of solitons still remain unchanged, but its transmission direction is deflected observed via Fig. 1a, c. Furthermore, when the imaginary part value increases, the angle between the transmission direction and x-axis increases. According to Fig. 1a, d, the real part of η has a certain impact on peak value, waveform and peak position. The larger the value of η is, the larger the peak value is. As the effect of α_2 and α_3 on one-soliton is not obvious, it will not be discussed here.

3.2 Two-soliton solution parameter analysis

Since ϕ_0 affects the peak position of solitons, it can be speculated that different values of ϕ_1, ϕ_2 can change the relative position between two solitons during their interactions. From Fig. 2, as the value of ϕ_1 increases, the initial position of the corresponding pulse moves. So the distance between two rows of solitons decreases, and the interaction of adjacent solitons becomes more fierce. When ϕ_1 increases to a certain extent, the interval between two solitons increases with the intensity of interaction decreasing. However, there is no change in the number of wave peaks in the unit period, so that the period of interaction cannot be adjusted by controlling ϕ_1. The parameter p_1, which is the real part of the parameter η, affects the peak position, peak value and waveform in the one-soliton propagation. And the imaginary part of the parameter p_2 affects the transmission direction of the one-soliton solution. Therefore, when studying the influence of parameter η on the interactions between two solitons, the real part and imaginary part are also separately discussed. According to Fig. 3, it is obvious that due

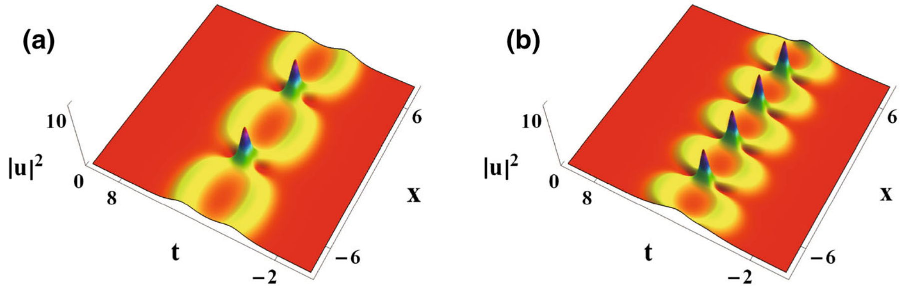

Fig. 4 Soliton interactions described by two-soliton solutions (21) with $\eta_2 = 1.5, \alpha_2 = 1, \alpha_3 = 0.01, \phi_2 = ln(0.5), \phi_1 = 0$. **a** $\eta_1 = 1 + 0.1I$. **b** $\eta_1 = 1 + 0.5I$

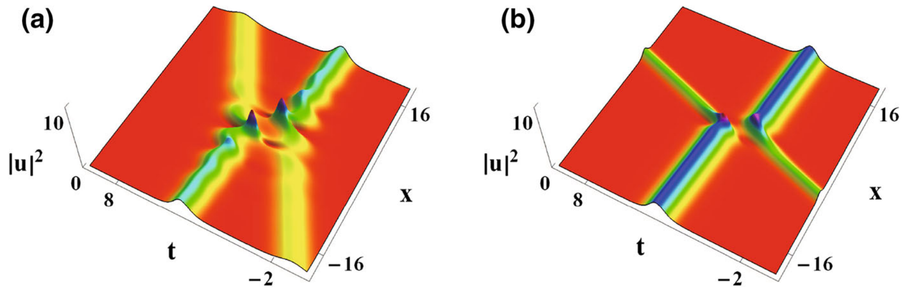

Fig. 5 Soliton interactions described by two-soliton solutions (21) with $\eta_1 = 1$, $\eta_2 = 1.5, \alpha_3 = 0.01, \phi_2 = ln(0.5), \phi_1 = 0$. **a** $\alpha_2 = 0.8$. **b** $\alpha_2 = 1.5$

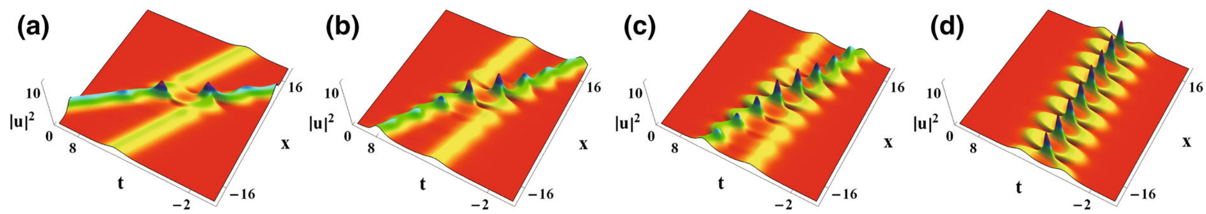

Fig. 6 Soliton interactions described by two-soliton solutions (21) with $\eta_1 = 1, \eta_2 = 1.5, \alpha_2 = 1, \phi_2 = ln(0.5), \phi_1 = 0$. **a** $\alpha_3 = 0.2$. **b** $\alpha_3 = 0.1$. **c** $\alpha_3 = 0.05$. **d** $\alpha_3 = 0$

to the influence of the real part of parameter η on the position of the soliton, the distance between the solitons is reduced even coincidence and the interaction becomes more obvious. In the meantime, the peak value controlled by η_1 also increases correspondingly with the increase in p_1. As the distance between solitons shrinks and the peak energy increases, interaction is enhanced and the period of interaction shortens. So different waveforms have different periods of interactions.

Next, we will consider the imaginary part of parameter η_j. According to the result in the previous section, when the imaginary part does not equal to zero, two solitons are no longer parallel to each other but intersect at a certain angle. And taking different imaginary value will change the angle of intersection. According to Fig. 4a, b, two solitons interact at the intersection and then continue to transmit along their respective directions, maintaining the original peak value, waveform and direction. Not only that, when the imaginary part value increases, the angle between two pulses becomes larger.

For α_2, when we keep other parameters unchanged, it can be seen from Fig. 5 that α_2 just affects the number of crests in a unit period. In another word, α_2 will impact on the period and does not have influence on other properties of optical solitons. In Fig. 6, different values of α_3 are chosen to observe the effect on the interaction between solitons. As we can see in Fig. 6, as the value of α_3 decreases, the angle between two solitons decreases. Until α_3 equals to 0, the angle is also reduced to 0 and propagates parallel to the x-axis. The interaction range increases during the decrease in the angle.

4 Conclusion

In the present paper, analytic one- and two-soliton solutions for the third-NLS equation have been obtained by the Hirota's bilinear method. The influences of each parameter on soliton interactions have been analyzed in detail. The amplitude of the soliton has been only determined by the real part of η_j, and η_j also has a certain impact on the peak position of solitons. The peak of the soliton has become higher with the increase in the real value of η_j, while the relative distance of two solitons has increased. Besides, the period has been affected by η_j. The angle between two solitons has become larger as the imaginary part of η_j increased. ϕ_j has influences on the initial position of solitons. α_2 has been related to the period directly, and the peaks have become dense as α_2 increased. Finally, α_3 can be used to change the directions of the soliton transmission. Results indicated that interactions between periodic solitons have been controlled with corresponding parameters.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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