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# Chirped bright and double-kinked quasi-solitons in optical metamaterials with self-steepening nonlinearity

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## ABSTRACT

Considering the self-steepening effect in a metamaterial (MM) can significantly change its behaviour. We study the propagation of ultrashort pulses in nonlinear MMs that is governed by a generalized nonlinear Schrödinger equation with higher order effects such as pseudo-quintic nonlinearity and self-steepening effect. A class of chirped quasi-soliton solutions is obtained in the presence of the self-steepening term, and some of which are derived for the first time. The solutions comprise chirped bright quasi-solitons on a constant and zero background, kink and anti-kink quasi-solitons, and double-kink quasi-solitons. It is found that the nonlinear chirp associated with each of these waves is directly proportional to the intensity and its amplitude can be controlled by selecting the self-steepening and dispersion coefficients. Particular cases of chirp-free quasi-solitons are discussed. The conditions on MM parameters for the formation of these structures are also presented. The obtained results are important to explore much richer localized light pulses in MMs.

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## 1. Introduction

Nonlinear propagation of light pulses in optical fibres has attracted much more attention in recent years because of their extensive applications to telecommunication and ultrafast signal routing systems (1). In the picosecond regime, the dynamics of such nonlinear wave packets are usually described by the standard nonlinear Schrödinger equation (NLSE), which contains only the group velocity dispersion (GVD) and self-phase modulation (SPM). This model equation is derived from Maxwell's equations (2), and it has two distinct types of localized solutions, bright and dark soliton solutions, which are, respectively, existent in the anomalous and normal dispersion regimes (3).

To improve the capacity of high-bit-rate transmission systems, it is essential to use ultrashort pulses whose durations are shorter than 100 fs (4). In this case, the nonlinear susceptibility will produce higher order nonlinear effects like the Kerr dispersion (i.e. self-steepening), the delayed nonlinear response and even the third-order dispersion.

Recently, the studies of the effect of self-steepening nonlinearity on the known characteristics of solitons in optical fibre media have become a topic of growing interest (5, 6). Such a physically important effect, which is due to the intensity dependence of group velocity (7), causes an asymmetrical spectral broadening of the pulse and distort the waveforms. It can also develop an optical shock, perceived as an extremely sharp near edge (5). However, in the presence of both dispersion and the Kerr nonlinear effect, solitons can be found that are robust against the shock formation (8).

Compared with optical fibres, the study of soliton's dynamics in nonlinear metamaterials (MMs) with dispersive permittivity and dispersive permeability has received little attention. Noting that MMs are artificial structures that display properties beyond those available in naturally occurring materials (9). Particularly, the rich linear and nonlinear electromagnetic properties enable MMs to be potential candidates for stable soliton and other nonlinear phenomena (10). Recently, Yang et al. (11) studied the existence of quasi-solitons in MMs with

pseudo-quintic nonlinearity under the very specific condition of the vanishing self-steepening nonlinearity. As a particular result, they obtained three new types of exact bright, dark, bright-grey quasi-soliton solutions in the absence of self-steepening parameter.

But with the increasing intensity of the optical field and further shortening of the pulses up to the femtosecond duration ( $< 100$  fs), the investigation of the self-steepening term has become necessary in optical systems (8). Thus, it is important to investigate the quasi-solitons dynamics in MMs in the presence of this effect since the latter will essentially influence the physical features of propagating pulses.

In this paper, we study for the first time to our knowledge the propagation characteristics of bright, kink and double-kink quasi-solitons in nonlinear MMs in the presence of self-steepening effect. It is demonstrated that the self-steepening nonlinearity could make the quasi-solitons chirped, and the nonlinear chirp related to these structures is directly proportional to the pulse intensity. The particular case of vanishing the self-steepening term which gives rise to the chirp-free bright quasi-soliton solutions is also discussed. Furthermore, we investigate the formation conditions and properties of these solutions in detail.

The paper is arranged as follows. In Section 2, we present the model under consideration and discuss its particular cases. In Section 3, we discuss the method that has been employed to get the chirped quasi-soliton solutions. In Section 4, we present the exact form of analytical solutions and discuss the particular case of obtaining the chirp-free quasi-solitons. Concluding remarks and perspectives are given in Section 5.

## 2. Theoretical model

We consider the propagation of ultrashort pulses in nonlinear MMs with non-Kerr-type nonlinearity. The evolution of complex envelope  $\psi(z, t)$  of the electric field can be described by the following generalized NLSE with higher order effects (11, 12):

$$i \frac{\partial \psi}{\partial z} + \frac{k_2}{2} \frac{\partial^2 \psi}{\partial t^2} + p_3 |\psi|^2 \psi - p_5 |\psi|^4 \psi - i s_1 \frac{\partial}{\partial t} (|\psi|^2 \psi) = 0, \quad (1)$$

where  $t = cT/\lambda_p$  and  $z = Z/\lambda_p$  are the respective normalized time and propagation distance, and  $\lambda_p$  is the plasma wavelength. Also  $k_2$  stands for the GVD coefficient,  $s_1$  represents the self-steepening coefficient, while  $p_3$  and  $p_5$  represent cubic and pseudo-quintic nonlinear coefficients, respectively.

Equation (1) contains many physically relevant particular cases. For instance, Equation (1) for  $p_5 = s_1 = 0$  reduces to the standard NLSE which includes only the GVD and SPM effects (13, 14). For  $p_5 = 0$ , Equation (1) represents the modified NLSE which governs the propagation of NLS soliton in the presence of Kerr dispersion (5). If setting  $p_3 = p_5 = 0$ , Equation (1) reduces to the Kaup–Newell equation which is usually called the derivative NLS I (15). Moreover, when  $s_1 = 0$ , Equation (1) reduces to the cubic–quintic NLSE which describes the wave dynamics in a non-Kerr medium exhibiting both third- and fifth-order susceptibilities (i.e.  $\chi^{(3)}$  and  $\chi^{(5)}$ ) (16). Furthermore, in case when  $p_3 = s_1 = 0$ , Equation (1) collapses to the case of quintic NLSE describing the wave propagation in a pure quintic nonlinear medium (17).

Considering the self-steepening effect to be negligible (i.e.  $s_1 = 0$ ), Yang et al. (11) have recently reported exact quasi-soliton solutions for this equation by using the ansatz method. Also, Marklund et al. (18) have investigated the stability properties of a pulse within the framework of Equation (1) and showed that the MM can influence the dynamics in significant ways. Our interest here is to obtain the exact chirped quasi-soliton solutions for this equation in the presence of self-steepening coefficient.

## 3. Description of the method

To derive the exact chirped quasi-soliton solutions of the above model, we use the following complex travelling-wave solution (19):

$$\psi(z, t) = \rho(\xi) e^{i[\kappa z - \Omega t - \phi(\xi)]}, \quad (2)$$

where  $\rho(\xi)$  is the unknown amplitude function assumed to be real,  $\xi = t - \chi z$  is the travelling coordinate,  $\phi(\xi)$  is the parameter of the phase modification, and the real parameters  $\chi$ ,  $\kappa$  and  $\Omega$  denote, respectively, the inverse velocity, wave number and frequency of the wave oscillation.

This envelope solution acquires an extra instantaneous frequency shift (i.e. chirp) given by

$$\delta\omega(z, t) = -\frac{\partial}{\partial t} [\kappa z - \Omega t - \phi(\xi)] = \Omega + \phi'(\xi), \quad (3)$$

where the prime represents differentiation with respect to  $\xi$ .

Substituting Equation (2) into Equation (1), the real and imaginary parts of the resulting equation, respectively, read

$$-(\kappa + \chi\phi')\rho + \frac{k_2}{2}\rho'' - \frac{k_2}{2}(\Omega + \phi')^2\rho + p_3\rho^3 - p_5\rho^5 - s_1(\Omega + \phi')\rho^3 = 0 \quad (4)$$

and

$$-\chi\rho' - k_2(\Omega + \phi')\rho' - \frac{k_2}{2}\rho\phi'' - 3s_1\rho^2\rho' = 0. \quad (5)$$

For solving the above pair of coupled equations, we take the ansatz of the form

$$\phi' = \alpha\rho^2 + \beta, \quad (6)$$

where  $\alpha$  and  $\beta$  denote the nonlinear and constant chirp parameters, respectively. The substitution of ansatz (6) into Equation (5) yields two algebraic equations that define the chirp parameters:

$$\alpha = -\frac{3s_1}{2k_2}, \quad \beta = -\Omega - \frac{\chi}{k_2}. \quad (7)$$

Accordingly, the resultant chirp consisting of linear and nonlinear contributions can be readily obtained as

$$\delta\omega(z, t) = -\frac{3s_1}{2k_2}\rho^2 - \frac{\chi}{k_2}. \quad (8)$$

We notice that the resultant chirp related to optical quasi-solitons is directly proportional to the pulse intensity (with  $I = |\psi|^2 = \rho^2$ ). We also see that the nonlinear chirp parameter depends essentially on self-steepening and GVD parameters. This implies that the amplitude of chirping can be controlled by selecting these coefficients.

Further substitution of expression (6) along with relations (7) into (4) gives

$$\rho'' + \delta\rho + \sigma\rho^3 + \gamma\rho^5 = 0, \quad (9)$$

where

$$\delta = \frac{\chi^2 + 2k_2(\Omega\chi - \kappa)}{k_2^2}, \quad (10a)$$

$$\sigma = \frac{2(p_3k_2 + s_1\chi)}{k_2^2}, \quad (10b)$$

$$\gamma = \frac{3s_1^2 - 8p_5k_2}{4k_2^2}. \quad (10c)$$

Equation (9) is an elliptic-type differential equation describing the evolution of the wave amplitude in the MM. As we all know, this equation can be mapped into  $\varphi^6$  field equation, which is well known to admit bright soliton, dark soliton, kink and double-kink solutions (20–22). In the following, we present different forms of quasi-soliton solutions for this equation, illustrating the richness of dynamics in nonlinear MMs governed by the model under consideration. The chirping related to these structures is also reported.

## 4. Results and discussions

Here we give exact analytical chirped quasi-soliton solutions of NLSE (1) in the fs region. It will be shown that these structures have nontrivial phase chirping which varies as a function of intensity due to the Kerr dispersion term.

### 4.1. Bright quasi-solitons

Here we present three types of chirped bright quasi-soliton solutions of Equation (1) describing quasi-soliton pulse on a constant and zero background.

#### 4.1.1. Type-I

The first class of quasi-soliton that a non-Kerr MM can support is of the form

$$\rho(\xi) = \frac{A \operatorname{sech}(\eta\xi)}{\sqrt{1 + B \operatorname{sech}^2(\eta\xi)}}, \quad (11)$$

which is an exact bright-type solution of Equation (9) with the following pulse parameters:

$$A = \sqrt{\frac{12B(1+B)(p_3k_2 + s_1\chi)}{(1+2B)(8p_5k_2 - 3s_1^2)}}, \quad (12a)$$

$$\eta = A \sqrt{\frac{p_3k_2 + s_1\chi}{(1+2B)k_2^2}}, \quad (12b)$$

$$\kappa = \frac{k_2}{2} \left[ \eta^2 - \Omega^2 + \left( \Omega + \frac{\chi}{k_2} \right)^2 \right]. \quad (12c)$$

Here the frequency shift  $\Omega$  and the inverse velocity  $\chi$  are arbitrary constants. Clearly, we must require that  $B(1+2B)(p_3k_2 + s_1\chi)(8p_5k_2 - 3s_1^2) > 0$  and  $(1+2B)(p_3k_2 + s_1\chi) > 0$ , in order to ensure real amplitude and inverse width relations, with the necessary conditions  $B > -1$  and  $B \neq -\frac{1}{2}$ .

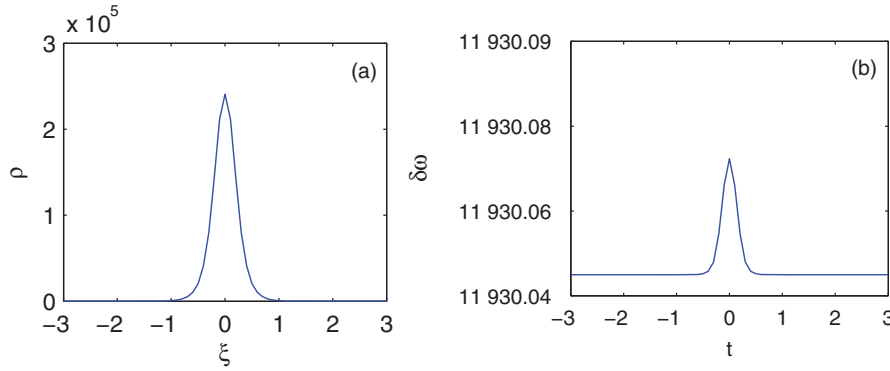
Based upon the above finding, we obtain a first nonlinearly chirped bright quasi-soliton solution of Equation (1) in the form

$$\psi(z, t) = \frac{A \operatorname{sech}[\eta(t - \chi z)]}{\sqrt{1 + B \operatorname{sech}^2[\eta(t - \chi z)]}} e^{i[\kappa z - \Omega t - \phi(\xi)]}, \quad (13)$$

where the soliton parameters  $A$ ,  $\eta$  and  $\kappa$  obey relations (12), while the frequency shift  $\Omega$  and the inverse velocity  $\chi$  are arbitrary constants.

The corresponding chirping is given by

$$\delta\omega(z, t) = -\frac{3s_1A^2 \operatorname{sech}^2(\eta\xi)}{2k_2[1 + B \operatorname{sech}^2(\eta\xi)]} - \frac{\chi}{k_2}. \quad (14)$$



**Figure 1.** Typical amplitude profile of the (a) chirped bright quasi-soliton solution given by Equation (11) and (b) chirping profile given by Equation (14) for  $B = 0.94$  and  $\chi = 2384.816$ .

Figure 1(a) depicts the amplitude profile of a typical chirped bright quasi-soliton for the following values of the model parameters (11):  $k_2 = -0.19990$ ,  $p_3 = 5.0265 \times 10^{-10}$  and  $p_5 = -2.095 \times 10^{-21}$ . The value of the self-steepening parameter is taken as  $s_1 = 6.2918 \times 10^{-14}$ . The corresponding chirping for the bright soliton is also shown in Figure 1(b) (for  $z = 0$ ). We clearly see that the chirp has a maximum at the centre of the wave and saturates at the same finite value as  $t \rightarrow \pm\infty$ .

**4.1.1.1. Chirp-free bright quasi-solitons (type-I).** It is easily seen that the particular exact bright quasi-soliton solution reported in (11) can be reproduced here. Take the case when the phase modification  $\phi(\xi) = 0$  in (2) for example; it follows from Equation (2) that the phase function reduces to a simple form  $\kappa z - \Omega t$ , which implies that the resulting envelope solution is a chirp-free quasi-soliton. By putting  $\phi(\xi) = 0$  in Equation (5), we obtain the relation  $\chi = -\Omega k_2$  and the condition  $s_1 = 0$ . Under the latter condition, the formulas in Equations (12) can be reduced to the form:

$$A = \sqrt{\frac{3B(1+B)p_3}{2(1+2B)p_5}}, \quad (15a)$$

$$\eta = A \sqrt{\frac{p_3}{(1+2B)k_2}}, \quad (15b)$$

$$\kappa = \frac{k_2}{2} (\eta^2 - \Omega^2). \quad (15c)$$

Then a direct substitution of these results in (13) yields a chirp-free solution for Equation (1) as follows:

$$\psi(z, t) = \frac{A \operatorname{sech}[\eta(t - \chi z)]}{\sqrt{1 + B \operatorname{sech}^2[\eta(t - \chi z)]}} e^{i(\kappa z - \Omega t)}, \quad (16)$$

which is exactly the bright quasi-soliton solution presented in (11). As can be clearly seen from (15a)

and (15b), the existence of this quasi-soliton structure requires  $B(1+2B)p_3p_5 > 0$  and  $(1+2B)k_2p_3 > 0$  with the condition  $B > -1$ .

#### 4.1.2. Type-II

We have found an exact bright-type solution of Equation (9) as

$$\rho(\xi) = \lambda \sqrt{1 + \operatorname{sech}(\eta\xi)}, \quad (17)$$

where

$$\lambda = \sqrt{-\frac{3(p_3k_2 + s_1\chi)}{3s_1^2 - 8p_5k_2}}, \quad (18a)$$

$$\eta = \lambda^2 \sqrt{\frac{3s_1^2 - 8p_5k_2}{3k_2^2}}, \quad (18b)$$

$$\kappa = \frac{4\chi(\chi + 2k_2\Omega)(8p_5k_2 - 3s_1^2) + 15(p_3k_2 + s_1\chi)^2}{8k_2(8p_5k_2 - 3s_1^2)}, \quad (18c)$$

provided that  $3s_1^2 - 8p_5k_2 > 0$  and  $p_3k_2 + s_1\chi < 0$ .

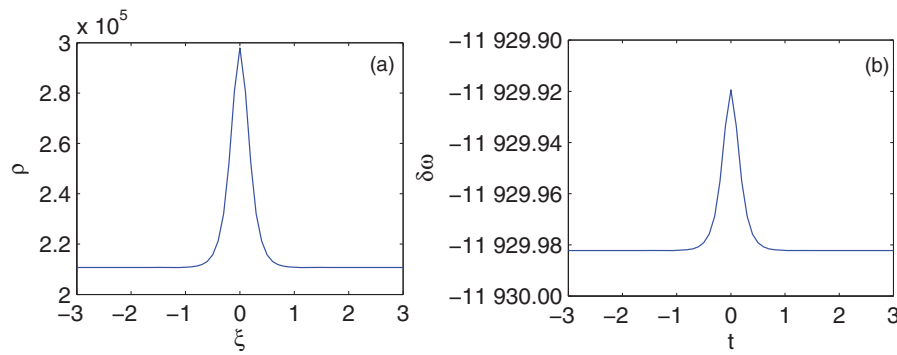
Having obtained the expressions for the pulse parameters  $\lambda$ ,  $\eta$  and  $\kappa$ , we can write the complete chirped bright quasi-soliton solution for Equation (1) as

$$\psi(z, t) = \lambda \sqrt{1 + \operatorname{sech}[\eta(t - \chi z)]} e^{i[\kappa z - \Omega t - \phi(\xi)]}, \quad (19)$$

and the chirp is given by

$$\delta\omega(z, t) = -\frac{3s_1}{2k_2} \lambda^2 (1 + \operatorname{sech}(\eta\xi)) - \frac{\chi}{k_2}. \quad (20)$$

The typical profiles for the amplitude and chirping (for  $z = 0$ ) are shown in Figures 2(a,b), respectively, for  $k_2 = -0.19990$ ,  $p_3 = -5.0265 \times 10^{-10}$ ,  $p_5 = 2.095 \times 10^{-21}$  and  $s_1 = 6.2918 \times 10^{-14}$ . It is clear from Figure 2(a) that the chirped bright quasi-soliton appears on a constant background in contrast to Figure 1(a).



**Figure 2.** Typical amplitude profile of the (a) chirped bright quasi-soliton solution given by Equation (17) and (b) chirping profile given by Equation (20) for  $\chi = -2384.816$ .

**4.1.2.1. Chirp-free bright quasi-solitons (type-II).** In the case of  $\phi(\xi) = 0$  in (2), Equation (5) yields the relation  $\chi = -\Omega k_2$  and the condition  $s_1 = 0$ . Under these conditions, Equation (19) simplifies to the chirp-free bright quasi-soliton solution:

$$\psi(z, t) = \lambda \sqrt{1 + \operatorname{sech}[\eta(t - \chi z)]} e^{i(\kappa z - \Omega t)}, \quad (21)$$

with the following parameters

$$\lambda = \sqrt{\frac{3p_3}{8p_5}}, \quad (22a)$$

$$\eta = \lambda^2 \sqrt{-\frac{8p_5}{3k_2}}, \quad (22b)$$

$$\chi = -\Omega k_2, \quad (22c)$$

$$\kappa = \frac{32p_5\chi(\chi + 2k_2\Omega) + 15p_3^2k_2}{64p_5k_2}. \quad (22d)$$

Necessary conditions for the existence of this solution can be obtained from Equations (22a) and (22b) as  $p_3p_5 > 0$  and  $p_5k_2 < 0$ .

**4.1.3. Type-III**

In the limit of  $\delta = 0$ , that is, at a specific value of the wave number  $\kappa$  given by  $\kappa = (1/2k_2)(\chi^2 + 2k_2\Omega\chi)$ , one obtains an interesting algebraic bright-type solution for (9) of the form

$$\rho(\xi) = \frac{1}{\sqrt{M + N\xi^2}}, \quad (23)$$

where

$$M = -\frac{3s_1^2 - 8p_5k_2}{12(p_3k_2 + s_1\chi)}, \quad (24a)$$

$$N = -\frac{(p_3k_2 + s_1\chi)}{k_2^2}, \quad (24b)$$

with the necessary conditions  $(3s_1^2 - 8p_5k_2) > 0$  and  $(p_3k_2 + s_1\chi) < 0$ .

Making use of these findings, we can present the third family of chirped bright quasi-soliton solution of Equation (1) as

$$\psi(z, t) = \frac{1}{\sqrt{M + N\xi^2}} e^{i[\kappa z - \Omega t - \phi(\xi)]}. \quad (25)$$

For this case, the chirping can be written as

$$\delta\omega(z, t) = -\frac{3s_1}{2k_2} \frac{1}{(M + N\xi^2)} - \frac{\chi}{k_2}. \quad (26)$$

The typical profiles for amplitude and chirping (for  $z=0$ ) are shown in Figures 3(a,b), respectively, for  $k_2 = -0.19990$ ,  $p_3 = -5.0265 \times 10^{-10}$ ,  $p_5 = 2.095 \times 10^{-21}$ ,  $s_1 = 6.2918 \times 10^{-14}$  and  $\chi = -2384.816$ .

**4.1.3.1. Chirp-free bright quasi-solitons (type-III).**

If we set  $\phi(\xi) = 0$  in (2), then Equation (5) yields the relation  $\chi = -\Omega k_2$  and the condition  $s_1 = 0$ . Considering the latter condition, it is easy to find that quasi-soliton solution (25) may be reduced to the following chirp-free bright quasi-soliton solution:

$$\psi(z, t) = \frac{1}{\sqrt{M + N\xi^2}} e^{i(\kappa z - \Omega t)}, \quad (27)$$

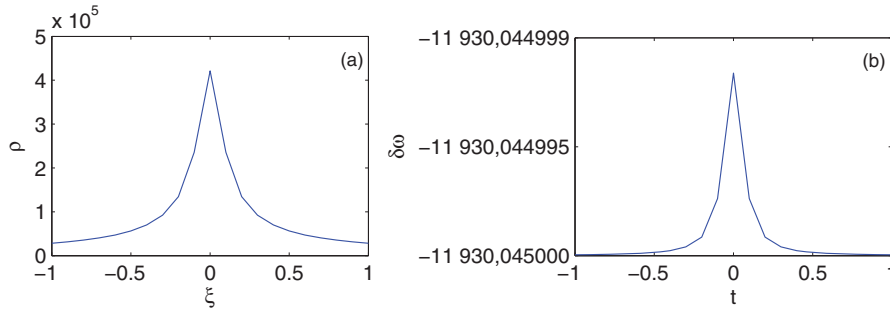
where

$$M = \frac{2p_5}{3p_3}, \quad N = -\frac{p_3}{k_2} \quad (28)$$

provided  $p_3p_5 > 0$  and  $p_3k_2 < 0$ .

**4.2. Kinked quasi-solitons**

In this subsection, we present the exact chirped kink quasi-soliton solutions and their characteristics for



**Figure 3.** Typical amplitude profile of the (a) chirped bright quasi-soliton solution given by Equation (23) and (b) chirping profile given by Equation (26) for the values mentioned in the text.

model equation (1). Note that kink-type solutions are waveforms with  $|\psi(z, t)|$  approaching zero at one end (the low end) and a nonzero constant at the other (high) end. In optical contexts, a kink soliton is a shock wave which propagates undistorted in a dispersive nonlinear medium (23). Generally, optical kink solitons are of both fundamental and technological importance if they are stable under propagation. But the occurrence of these structures in nonlinear optics is relatively rare (24).

In what follows, kink quasi-soliton solutions are reported for the first time in the setting of MMs with dispersive permittivity and dispersive permeability. In particular, we consider the case in which the coefficient  $\gamma$  in amplitude equation (9) is related to  $\delta$  and  $\sigma$  with the relation  $\gamma = 3\sigma^2/16\delta$ . This corresponds to a specific value of the wave number given by

$$\kappa = \frac{(\chi^2 + 2k_2\Omega\chi)(3s_1^2 - 8p_5k_2) - 3(p_3k_2 + s_1\chi)^2}{2k_2(3s_1^2 - 8p_5k_2)}. \quad (29)$$

In this case, we find that Equation (9) admits exact solutions of the form

$$\rho(\xi) = Q\sqrt{1 \pm \tanh(\mu\xi)}, \quad (30)$$

which describes kink-type (upper sign) and anti-kink-type (lower sign) solution with the parameters

$$Q = \sqrt{\frac{\chi^2 + 2k_2(\Omega\chi - \kappa)}{(p_3k_2 + s_1\chi)}}, \quad (31a)$$

$$\mu = \sqrt{\frac{\chi^2 + 2k_2(\Omega\chi - \kappa)}{k_2^2}}, \quad (31b)$$

provided  $\chi^2 + 2k_2(\Omega\chi - \kappa) < 0$  and  $p_3k_2 + s_1\chi > 0$ .

Hence, we obtain a chirped kink and anti-kink quasi-soliton solution of Equation (1) as

$$\psi(z, t) = Q\sqrt{1 \pm \tanh[\mu(t - \chi z)]}e^{i[\kappa z - \Omega t - \phi(\xi)]}, \quad (32)$$

for which the chirping will be

$$\delta\omega(z, t) = -\frac{3s_1}{2k_2}Q^2[1 \pm \tanh(\mu\xi)] - \frac{\chi}{k_2}. \quad (33)$$

In Figures 4(a,b), the typical profiles for the amplitude and chirping (for  $z=0$ ) are shown for the model parameters (11):  $k_2 = -0.7954$ ,  $p_3 = 1.2566 \times 10^{-10}$  and  $p_5 = -2.095 \times 10^{-21}$ . The self-steepening parameter is taken as  $s_1 = 6.2918 \times 10^{-14}$  while  $\chi = 3336.88$  and  $\Omega = 0$ .

**4.2.0.2. Chirp-free kinked quasi-solitons.** If we set  $\phi(\xi) = 0$  in (2), then Equation (5) yields the relation  $\chi = -\Omega k_2$  and the condition  $s_1 = 0$ . In this case, Equation (23) simplifies to the following chirp-free kink and anti-kink quasi-soliton solutions:

$$\psi(z, t) = Q\sqrt{1 \pm \tanh[\mu(t - \chi z)]}e^{i(\kappa z - \Omega t)} \quad (34)$$

where

$$Q = \sqrt{\frac{\chi^2 + 2k_2(\Omega\chi - \kappa)}{p_3k_2}}, \quad (35a)$$

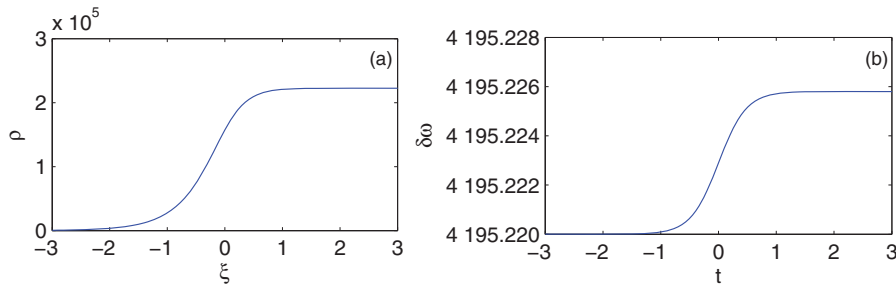
$$\mu = \sqrt{\frac{\chi^2 + 2k_2(\Omega\chi - \kappa)}{k_2^2}}, \quad (35b)$$

provided that  $\chi^2 + 2k_2(\Omega\chi - \kappa) < 0$  and  $p_3k_2 > 0$ .

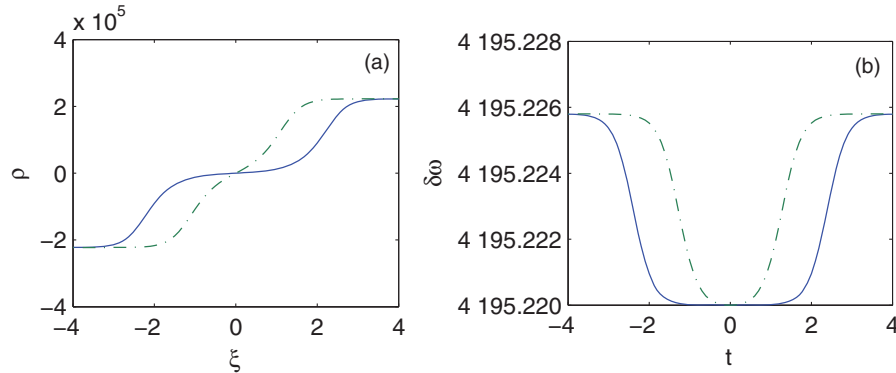
### 4.3. Double-kinked quasi-solitons

Equation (9) possesses double-kink solutions of the form (21)

$$\rho(\xi) = \frac{p \sinh(\mu\xi)}{\sqrt{\epsilon + \sinh^2(\mu\xi)}}, \quad (36)$$



**Figure 4.** Typical amplitude profile of the (a) chirped kink quasi-soliton solution given by Equation (30) and (b) chirping profile given by Equation (33) for the values mentioned in the text.



**Figure 5.** Typical amplitude profile of the (a) chirped double-kink quasi-soliton solution given by Equation (36) and (b) chirping profile given by Equation (39) for the values mentioned in the text. Here the values of  $\epsilon$  are  $\epsilon = 5000$  (solid line) and  $\epsilon = 50$  (dashed line).

where

$$\mu = \sqrt{-\frac{\delta\epsilon}{\epsilon - 3}}, \quad (37a)$$

$$p = \sqrt{-\frac{2\delta(2\epsilon - 3)}{\sigma(\epsilon - 3)}}, \quad (37b)$$

$$\gamma = -\frac{3\mu^2}{p^4} \left( \frac{\epsilon - 1}{\epsilon} \right) \quad (37c)$$

provided that  $\epsilon \neq 3$ ,  $\epsilon \neq \frac{3}{2}$  and  $\epsilon$  should take sufficiently large values in order to get double-kink-type quasi-soliton. Notice that reality of  $\mu$  and  $p$  in (37a) and (37b) can be ensured by demanding negativity of  $\delta$  and positivity of  $\sigma$  (namely,  $\delta < 0$  and  $\sigma > 0$ ).

For this case, the complete chirped quasi-soliton solution of Equation (1) will be of the following form:

$$\psi(z, t) = \frac{p \sinh[\mu(t - \chi z)]}{\sqrt{\epsilon + \sinh^2[\mu(t - \chi z)]}} e^{i[kz - \Omega t - \phi(\xi)]}, \quad (38)$$

and the chirping can be written as

$$\delta\omega(z, t) = -\frac{3s_1 p^2 \sinh^2(\mu\xi)}{2k_2 [\epsilon + \sinh^2(\mu\xi)]} - \frac{\chi}{k_2}. \quad (39)$$

The amplitude profile of this chirped quasi-soliton solution is illustrated in Figure 5(a) for the values of parameters (11):  $k_2 = -0.7954$ ,  $p_3 = 1.2566 \times 10^{-10}$  and  $p_5 = -2.095 \times 10^{-21}$ . The self-steepening parameter is taken as  $s_1 = 6.2918 \times 10^{-14}$  while  $\chi = 3336.88$  and  $\Omega = 0$ . We can see that it exhibits a double-kink structure for  $\epsilon = 5000$ , while the two kinks join into a single one for  $\epsilon = 50$ . Figure 5(b) illustrates the chirp associated with this interesting envelope double-kink quasi-soliton.

It is worth noting that many other types of functional forms describing chirpedbright-type quasi-solitons can also be obtained for dynamical amplitude equation (9) and consequently for generalizedNLSE (1). This indicates the richness of dynamics in nonlinear MM governed by Equation (1).

A remarkable result is that all parameters of the obtained chirped quasi-solitons depend on the self-steepening coefficient  $s_1$ . Hence, this effect may significantly influence the properties of these wave packets. We also see that the self-steepening nonlinearity is an important parameter for the formation of the derived chirped quasi-solitons, and has an impact on the nonlinear chirp parameter associated with each of these waves. In contrast, the pseudo-quintic nonlinearity has no effect on the nonlinear chirp parameter. It only influences the amplitude, inverse pulse and the wave number of quasi-solitons.



## 5. Conclusions

In conclusion, we derived exact chirped quasi-soliton solutions of a generalized NLSE describing the propagation of ultrashort pulses in nonlinear (non-Kerr-type) MMs. Three types of chirped bright quasi-soliton solutions of this equation were found in the presence of self-steepening effect by employing the travelling-wave method. We also presented the chirped kink, anti-kink and double-kink quasi-soliton solutions of the model for some constraints. Such privileged structures characteristically exist due to a balance among the GVD, pseudo-quintic nonlinearity and self-steepening effect. The chirp related to these localized wave packets has also been identified and found that it is directly proportional to the intensity of the wave. We have further deduced the exact chirp-free quasi-soliton solutions of the model under the condition of the vanishing self-steepening parameter. The amplitude and chirp profiles of these quasi-soliton solutions are also shown for specific values of MM parameters.

Interesting subjects for future research would include systematic studies of the stability of such chirped femtosecond quasi-solitons under some perturbations by employing the numerical simulation methods. It may be amplitude perturbation, random noises and the slight violation of the parametric conditions. Detailed stability analyses are now under investigation.

## Disclosure statement

No potential conflict of interest was reported by the authors.

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