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Bright, dark and W-shaped solitons with extended nonlinear Schrödinger's equation for odd and even higher-order terms

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ABSTRACT

We present solitary wave solutions of an extended nonlinear Schrödinger equation with higher-order odd (third-order) and even (fourth-order) terms by using an ansatz method. The including high-order dispersion terms have significant physical applications in fiber optics, the Heisenberg spin chain, and ocean waves. Exact envelope solutions comprise bright, dark and W-shaped solitary waves, illustrating the potentially rich set of solitary wave solutions of the extended model. Furthermore, we investigate the properties of these solitary waves in nonlinear and dispersive media. Moreover, specific constraints on the system parameters for the existence of these structures are discussed exactly. The results show that the higher-order dispersion and nonlinear effects play a crucial role for the formation and properties of propagating waves.

1. Introduction

In recent years, considerable attention has been paid theoretically and experimentally to analyze the propagation of ultrashort pulses in optical fibers. For picosecond pulses, the wave dynamics is governed by the well known nonlinear Schrödinger (NLS) equation that includes the group velocity dispersion (GVD) and self-phase modulation (SPM) [1,2]. This model equation is completely integrable by the inverse scattering transform (IST) [3], and it has two distinct types of localized solutions, bright and dark soliton solutions, which are, respectively, existent in the anomalous and normal dispersion regimes [4–7]. Physically, such structures exist as a result of the balance between GVD and SPM nonlinearity. It is worth noting that the SPM is the nonlinear effect due to the lowest dominant nonlinear susceptibility $\chi^{(3)}$in silica fibers [8].

However, as one increases the intensity of the incident light power to produce shorter (femtosecond) pulses, the standard NLS equation becomes inadequate to describe the wave dynamics more accurately. Some additional higher-order effects, such

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as third- and fourth-order dispersions, self-steepening, and nonlinear response effects, will play important roles in the propagation of optical pulses. In particular, it has been demonstrated that if short pulses whose width is nearly 50 fs have to be injected; the third-order dispersion becomes important and must be included in the governing equation [9–11]. Moreover, as the pulses become extremely short (below 10 fs), fourth-order dispersion also must be taken into account [12,13]. In addition, it was found that a relatively large number of optical materials display higher-order nonlinear effects such the non-Kerr quintic nonlinearity arising due to fifth-order susceptibility$\chi^{(5)}$. Among them, we mention here semiconductors (e.g., $Al_xGa_{1-x}As$, CdS, and $CdS_{1-x}Se_x$) waveguides and semiconductor-doped glasses (see, e.g. [14]). In addition, it has been shown that cubic-quintic nonlinearity correctly describes the dielectric response of poly diacetylene p-toluene sulphonate (PTS) crystal [15–17]. Theoretically, the quintic nonlinearity occurs from the expansion of the refractive index in powers of intensity I of the light pulse as: $n = n_0 + n_2 I + n_4 I^2 + \ldots$, Here n_0 is the linear refractive index coefficient, n_2 and n_4 are the cubic and quintic nonlinearity coefficients which are related to third-order and fifth-order susceptibility, respectively [18].

To describe the propagation of femtosecond pulses in optical fibers with the above mentioned effects, one has to extend the order of nonlinear terms in the standard NLS equation beyond Kerr nonlinearity. The higher-order nonlinear Schrödinger (HNLS) equation with additional correction terms is therefore the main nonlinear equation governing the pulse evolution in highly nonlinear media. Note that with the consideration of higher-order effects, the physical features and the stability of the NLS soliton can change significantly.

A number of generalizations of the cubic NLS equation have been proposed to describe nonlinear propagation of ultrashort pulses in a variety of nonlinear dispersive materials, which may allow a more accurate adjustment of the usual equation to practical needs. Well known higher-order NLS models include the Hirota [19], Sasa-Satsuma [20], and Lakshmanan-Porsezian-Daniel (LPD) [21] equations. As concerns the LPD equation, this important model is the result of a generalization of the cubic NLS equation that includes a fourth-order dispersion term.

More recently, Ankiewicz et al. proposed a further generalization of the NLS equation, adding a third-order dispersion term to the LPD equation of the form [22]:

$$iE_x + \frac{1}{2}E_{tt} + |E|^2E + \gamma\left(E_{tttt} + 6r_1E_t^2E^* + 4r_2E|E_t|^2 + 8r_3E_{tt}|E|^2 + 2r_4E_{tt}^*E^2 + 6r_5E|E|^4\right) - i\alpha_3\left(E_{ttt} + 6r_6E_t|E|^2 + 6r_7E^2E_t^*\right) = 0 \tag{1}$$

where $E(x,t)$ represents the envelope of the waves in the context of nonlinear optics, x is the propagation variable, and t is time in the moving frame. The subscripts indicate partial derivatives. The seven real constants r_i $(i = 1, \ldots, 7)$ are free real parameters, and the coefficients α_3 and γ are also real parameters related to third-order dispersion E_{ttt} and fourth-order dispersion E_{tttt}, respectively. Also, the terms $|E|^2E$ and $|E|^4E$ represent cubic and quintic nonlinearities of the medium, respectively. The other higher-order terms in Eq. (1) are needed for sufficiently short pulses.

Equation (1) is very general because it includes many integrable particular cases. In the absence of third- and fourth-order dispersion terms ($\alpha_3 = \gamma = 0$), the model equation reduces to the standard NLSE which has only the terms describing lowest order dispersion and self-phase modulation. Moreover, for$\gamma = 0$,$r_6 = 1$ and$r_7 = 0$, Eq. (1) becomes the Hirota equation [19], while the case $\alpha_3 = 0$ and$r_j = 1$ (with $j = 1, .., 5$) gives rise the LPD equation [21]. Additionally, the Sasa-Satsuma equation [22] corresponds to the particular when$\gamma = 0$,$r_6 = 3/2$ and$r_7 = 1/2$. Thus, due to these relevant integrable cases, Eq. (1) is capable of describing the wave propagation in a variety of physical systems. In the most general case, when all the parameters are nonzero, i.e., when $\alpha_3 \neq 0$, $\gamma \neq 0$, and $r_i \neq 0$, exact rogue wave solutions of the first and second order have been presented in Ref. [22]. Furthermore, the existence of dipole soliton solutions of Eq. (1) has recently been demonstrated under specific conditions [23]. Besides, a derivation of soliton solutions and rogue wave of Eq. (1) with $r_7 = 0$ and$r_j = 1$ (with $j = 1, .., 6$) appears in Refs. [24,25]. The existence of N-soliton solutions in this extended model has been recently investigated by Yomba and Zakeri [26]. As a matter of fact, this model is very important especially to adjust to the current progress in highly dispersive nonlinear fiber media.

It is desirable to study the influence of additional higher-order dispersion terms, along with the quintic non-Kerr nonlinearity and higher-order nonlinear dispersive terms on the properties of different types of exact solutions of Eq. (1), e.g., W-shaped solution, bright and dark optical solitary wave solutions. Certainly, such investigation is relevant for a better understanding of the mechanism of the complicated physical phenomena and dynamical processes modeled by the model. We mention that having an explicit analytic solution has the advantage that we can compare experimental results with theory.

In this paper, we are going to employ the complex amplitude function ansatz proposed by Li et al. [27] to investigate exact solitary wave solutions for the extended model (1). Note that such an ansatz has been successfully utilized to derive solitary wave solutions for large classes of higher-order NLS family of equations [28–30]. The solutions comprise solitary wave structures of the bright, dark and W-shaped kind. We also study the properties and evolution of the solitary wave envelopes and find the parametric conditions of their existence.

The rest of the paper is organized as follows. In Sec. II, we present the extended NLS equation with additional third- and fourth-order dispersive terms, which are of particular importance in diverse nonlinear physical systems, such as in fiber optics, in the case of the Heisenberg spin chain, and for ocean waves. In Sec. III, we derive solitary wave solutions of the model equation by adopting a complex amplitude ansatz. Parametric conditions for the existence of obtained solutions along with the wave properties are also presented. Finally, our conclusions will be addressed in Sec. IV.

2. Theoretical background

To obtain the exact solitary wave solutions of Eq. (1) we consider a solution of the form [27].

$$E(x,t) = A(x,t)\exp[i\varphi(x,t)] \tag{2}$$

where $A(x,t)$is the complex envelope function and $\varphi(x,t) = kx - \Omega t$ is the linear phase shift. Also, k and $\Omega$are the respective real parameters describing wave number and frequency shift. Substituting Eq. (2) into Eq. (1) and removing the exponential term, we recast Eq. (1) as:

$$iA_x + ia_1A_t + a_2A_{tt} + a_3|A|^2A - ia_4|A|^2A_t - ia_5A^2A_t^* - a_6A + a_7|A|^4A + \gamma A_{tttt} - ia_8A_{ttt} + a_9A_t^2A^* + a_{10}|A_t|^2A + a_{11}|A|^2A_{tt} + a_{12}A^2A_{tt}^* = 0 \tag{3}$$

where

$$a_1 = -\Omega + 3\alpha_3\Omega^2 + 4\gamma\Omega^3$$

$$a_2 = \frac{1}{2} - 3\alpha_3\Omega - 6\gamma\Omega^2$$

$$a_3 = 1 + 6\alpha_3\Omega(r_7 - r_6) + 2\gamma\Omega^2(2r_2 - 3r_1 - 4r_3 - r_4)$$

$$a_4 = 6\alpha_3r_6 + 4\gamma\Omega(3r_1 - r_2 + 4r_3)$$

$$a_5 = 6\alpha_3r_7 + 4\gamma\Omega(r_2 - r_4)$$

$$a_6 = k + \frac{1}{2}\Omega^2 - \alpha_3\Omega^3 - \gamma\Omega^4$$

$$a_7 = 6\gamma r_5$$

$$a_8 = \alpha_3 + 4\gamma\Omega$$

$$a_9 = 6\gamma r_1$$

$$a_{10} = 4\gamma r_2$$

$$a_{11} = 8\gamma r_3$$

$$a_{12} = 2\gamma r_4$$

To search for exact solitary wave solutions of Eq. (3), we adopt a complex amplitude ansatz as follows [27]:

$$A(x,t) = i\beta + \lambda\tanh[\eta(t - \chi x)] + i\rho\,sech[\eta(t - \chi x)] \tag{4}$$

where $\eta$and$\chi$are the pulse width and shift of inverse group velocity, respectively. From the ansatz solution (4), it is apparent that the amplitude of solitary wave solutions is nonzero when the time variable approaches infinity.

In the limit$\beta = \lambda = 0$, we obtain a bright solitary wave solution, but when $\rho = 0$ the solution given in Eq. (4) transforms to dark solitary waveform. The presence of the parametersβ, λ and ρpermits the ansatz (4) to describe the features of both bright and dark solitary waves. It is worth noting that$\eta\chi$, k and $\Omega$are all real values but β, λ and ρcan be real or complex numbers [31].

Then we find the nontrivial non-linear phase shift function$\varphi(x,t)$as:

$$\varphi(x,t) = \arctan\left(\frac{\beta + \rho\,sech[\eta(t - \chi x)]}{\lambda\tanh[\eta(t - \chi x)]}\right) \tag{5}$$

Further, we can see that the amplitude function takes the form:

$$|A(x,t)| = \left\{\left(\lambda^2 + \beta^2\right) + 2\beta\rho\,sech[\eta(t - \chi x)] + \left(\rho^2 - \lambda^2\right)sech^2[\eta(t - \chi x)]\right\}^{1/2} \tag{6}$$

Substituting Eq. (4) into Eq. (3), expanding *tanh* terms to *sech* term and setting the coefficients for the independent terms containing independent combinations of this hyperbolic functions equal to zero, we obtain the following 11 independent parametric equations:

$$\lambda\left[6a_8\eta^3-\eta(a_4+a_5)\left(\rho^2-\lambda^2\right)\right]+\beta a_7\left(\rho^2-\lambda^2\right)\left(5\rho^2-\lambda^2\right)-a_9\beta\eta^2\left(\rho^2+\lambda^2\right)-a_{10}\beta\eta^2\left(\rho^2-\lambda^2\right)-4a_{11}\beta\eta^2\rho^2-4a_{12}\beta\eta^2\left(\rho^2-\lambda^2\right)=0 \tag{7a}$$

$$\rho\left[6a_8\eta^3-\eta(a_4+a_5)\left(\rho^2-\lambda^2\right)+4a_7\beta\lambda\left(\rho^2-\lambda^2\right)\right]-2\beta\lambda\rho\eta^2(a_9+2a_{11})=0 \tag{7b}$$

$$\rho\left[-2\eta^2a_2+a_3\left(\rho^2-\lambda^2\right)-2a_4\beta\lambda\eta-20\gamma\eta^4+a_9\eta^2\left(\rho^2-2\lambda^2\right)+a_{10}\eta^2\rho^2\right]+2a_7\rho\left[\beta^2\left(5\rho^2-3\lambda^2\right)+\lambda^2\left(\rho^2-\lambda^2\right)\right]+a_{11}\rho\eta^2\left[\left(\rho^2-\lambda^2\right)-2\left(\beta^2+\lambda^2\right)\right]+a_{12}\rho\eta^2\left[\left(\rho^2-\lambda^2\right)-2\beta^2\right]=0 \tag{7c}$$

$$\lambda\left[-2a_2\eta^2-8\gamma\eta^4+a_3\left(\rho^2-\lambda^2\right)-\rho^2\eta^2(a_9-a_{10})-2a_{11}\eta^2\left(\beta^2+\lambda^2\right)\right]-2\beta\eta\left[a_4\rho^2+a_5\left(\rho^2-\lambda^2\right)\right]+2a_7\lambda\left[\left(\beta^2+\lambda^2\right)\left(\rho^2-\lambda^2\right)+2\beta^2\rho^2\right]+2a_{12}\lambda\eta^2\left[\rho^2-\lambda^2+\beta^2\right]=0 \tag{7d}$$

$$\lambda\left[-\eta\chi+a_1\eta-4a_8\eta^3-a_4\eta\left(\beta^2+\lambda^2\right)-a_5\eta\left(\lambda^2-\beta^2-2\rho^2\right)\right]+a_3\beta\left(3\rho^2-\lambda^2\right)+2a_7\beta\left[\left(\beta^2+\lambda^2\right)\left(\rho^2-\lambda^2\right)+2\rho^2\left(2\beta^2+\lambda^2\right)\right]+\beta\rho^2\eta^2(a_9+a_{10}+2a_{11})+2a_{12}\beta\eta^2\left(\rho^2-2\lambda^2\right)=0 \tag{7e}$$

$$-\rho\left[\eta\chi-a_1\eta+a_8\eta^3-2a_3\beta\lambda+\lambda^2\eta(a_4-a_5)+\beta^2\eta(a_4+a_5)\right]+4a_7\beta\lambda\rho\left(\beta^2+\lambda^2\right)+2a_{12}\beta\lambda\rho\eta^2=0 \tag{7f}$$

$$\rho\left[\eta^2a_2+\eta^4\gamma+a_3\left(3\beta^2+\lambda^2\right)+2a_5\beta\eta\lambda-a_6+a_7\left(\beta^2+\lambda^2\right)\left(5\beta^2+\lambda^2\right)+a_{11}\eta^2\left(\beta^2+\lambda^2\right)+a_{12}\eta^2\left(\beta^2-\lambda^2\right)\right]=0 \tag{7g}$$

$$\lambda\left[a_3\left(\beta^2+\lambda^2\right)-a_6+a_7\left(\beta^2+\lambda^2\right)^2\right]=0 \tag{7h}$$

$$\beta\left[a_3\left(\beta^2+\lambda^2\right)-a_6+a_7\left(\beta^2+\lambda^2\right)^2\right]=0 \tag{7i}$$

$$\rho\left[24\eta^4\gamma+a_7\left(\rho^2-\lambda^2\right)^2-\eta^2\left(\rho^2-\lambda^2\right)(a_9+a_{10}+2a_{11}+2a_{12})\right]=0 \tag{7j}$$

$$\lambda\left[24\eta^4\gamma+a_7\left(\rho^2-\lambda^2\right)^2-\eta^2\left(\rho^2-\lambda^2\right)(a_9+a_{10}+2a_{11}+2a_{12})\right]=0 \tag{7k}$$

From the above 11 equations, we will discuss the various possible cases which enable us to distinguish the different types of solitary wave solutions that exist for the extended NLS equation with higher-order odd and even terms (1).

3. Exact solitary wave solutions and their properties

In this section, we impose some restrictions on the depending parameters so that the resulting Eqs. (7a)-(7k) become compatible. Solving these equations, we find that there exist three cases of exact solitary wave solutions for Eq. (1). In what follows, we present these solutions and investigate the formation conditions and properties of solitary waves in detail.

3.1. Case 1

When $a_4 + a_5 = 0$and $a_8 = a_{11} = a_{12} = 0$, we have found the solution of Eq. (3) as

$$A(x,t) = \mp\frac{1}{2}\rho + i\rho sech[\eta(t-\chi x)] \tag{8}$$

which implies that $\beta = \pm\frac{i}{2}\rho$and $\lambda = 0$ in the ansatz solution (4). Consequently, from Eqs. (7a)-(7k), we obtain the relations:

$$a_1 = \chi,\ a_2 = -5\gamma\eta^2,\ a_3 = -\frac{5\gamma\eta^4}{\rho^2},\ a_6 = \frac{13\gamma\eta^4}{8},\ a_7 = \frac{6\gamma\eta^4}{\rho^4},\ a_9 + a_{10} = \frac{30\gamma\eta^2}{\rho^2} \tag{9}$$

These results yield the following parameters of the solitary wave solution:

$$\eta^2 = -\frac{12\gamma\Omega^2 + 1}{10\gamma} \tag{10}$$

$$\chi = -8\gamma\Omega^3 - \Omega \tag{11}$$

$$k = \frac{13}{8}\gamma\eta^4 - \frac{1}{2}\Omega^2 - 3\gamma\Omega^4 \tag{12}$$

$$\rho^4 = \frac{\eta^4}{r_5} \tag{13}$$

With

$$\Omega = -\frac{\alpha_3}{4\gamma} \tag{14}$$

Combining Eqs. (2) and (8), we obtain the complete solution for Eq. (1) in the form

$$E(x,t) = \left\{\mp\frac{1}{2}\rho + i\rho sech[\eta(t-\chi x)]\right\}\exp[i(kx - \Omega\, t)] \tag{15}$$

and its corresponding intensity function is in the form

$$|E(x,t)|^2 = \rho^2\left[\frac{1}{4} + sech^2[\eta(t-\chi x)]\right] \tag{16}$$

From the above mentioned conditions, we find that the solitary wave solution (8) exists for Eq. (1) when$r_3 = r_4 = 0$ and $r_6 + r_7 = r_1/2$. One can also see from inserting (14) into (10) that the fourth-order dispersion coefficient must satisfy the condition $\gamma < -3\alpha_3^2/4$, which implies that the present solution exists for negative values of the fourth-order dispersion. Additionally, one must choose the parameter of quintic nonlinearity to satisfy$r_5 > 0$, as clearly seen in from Eq. (13). It is worth observing that here the pulse width η depends only on the third- and fourth-order dispersion parameters α_3 andγ [see Eq. (10) with (14)].

The intensity profile of the solitary wave solution is depicted in Fig. 1(a), as computed from Eq. (16) for the values $\rho = \eta = 1$ and$\chi = 0.1$. From it, one can clearly see that the solution (8) represents a bright solitary wave that propagates on a continuous wave background in the presence of higher-order effects. It is interesting to see that the wave profile remains unchanged during evolution which we have shown in Fig. 1(b).

3.2. Case 2

When $a_4 + a_5 = 0$and $a_7 = a_8 = a_{10} = 0$, one obtains a solitary wave solution of Eq. (3) of the form

$$A(x,t) = -\frac{\sqrt{5}}{4}i\rho + i\rho\sec h[\eta(t-\chi x)] \tag{17}$$

which means that $\beta = -\frac{\sqrt{5}}{4}\rho$and $\lambda = 0$in the ansatz solution (4). Accordingly, from Eqs. (7a)-(7k), we get the following expressions:

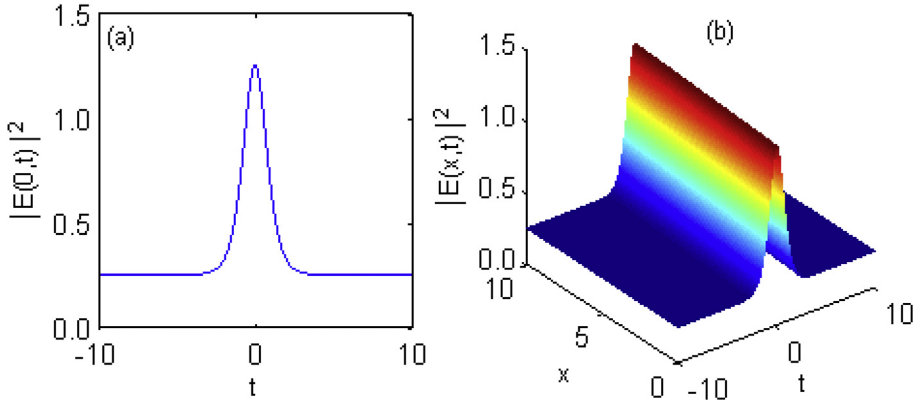

Fig. 1. (a) Intensity of the solitary wave profile $|E(0,t)|^2|$ as a function of t and its (b) evolution as computed from Eq. (16) for the values $\rho = \eta = 1$ and$\chi = 0.1$.

$$a_1 = \chi,\ a_2 = \frac{31}{4}\gamma\eta^2,\ a_3 = -\frac{8\gamma\eta^4}{\rho^2},\ a_6 = -\frac{5}{2}\gamma\eta^4,\ a_9 = \frac{48\gamma\eta^2}{\rho^2},\ a_{11} + a_{12} = -\frac{12\gamma\eta^2}{\rho^2} \tag{18}$$

These relations lead to the following solitary wave parameters:

$$\eta^2 = \frac{24\gamma\Omega^2 + 2}{31\gamma} \tag{19}$$

$$\chi = -8\gamma\Omega^3 - \Omega \tag{20}$$

$$k = -\frac{5}{2}\gamma\eta^4 - \frac{1}{2}\Omega^2 - 3\gamma\Omega^4 \tag{21}$$

$$\rho^2 = \frac{8\eta^2}{r_1} \tag{22}$$

With

$$\Omega = -\frac{\alpha_3}{4\gamma} \tag{23}$$

With these parameters, we find that the complete solitary wave solution for Eq. (1) can be written in the form

$$E(x,t) = \left\{ -\frac{\sqrt{5}}{4} i\rho + i\rho\, \sec h[\eta(t - \chi x)] \right\} \exp[i(kx - \Omega\, t)] \tag{24}$$

and its intensity is given by

$$|E(x,t)|^2 = \left[-\frac{\sqrt{5}}{4}\rho + \rho sech[\eta(t - \chi x)] \right]^2 \tag{25}$$

Bases on the above conditions, we find that the solution (17) exists provided that$r_1 > 0$, $r_2 = r_5 = 0$and$\gamma > -3\alpha_3^2/4$. Additionally, Eq. (23) together with the condition $a_4 + a_5 = 0$ require self-steepening and self-frequency shift coefficients must satisfy $r_6 + r_7 = (3r_1 + 4r_3 - r_4)/6$. Note that the wave parameter ρ is directly proportional to the width η, and depends only on the coefficient r_1 [see Eq. (22)].

Fig. 2(a) shows the intensity profile of the solitary wave solution as computed from Eq. (25) for the parameter values $\rho = \eta = 1$and$\chi = 0.1$, while Fig. 2(b) depicts its evolution. Unlike the first case, we see that the intensity profile of the wave takes the shape of W. It is worth noting that this type of solitary wave solutions has first been found for a higher-order NLS equation with third-order dispersions, self steepening, and self-frequency shift effects by Li et al. [25].

3.3. Case 3

When $a_4 - 3a_5 = 0$and $a_7 = a_9 = a_{11} = 0$, one can find a solitary wave solution of Eq. (3) of the form

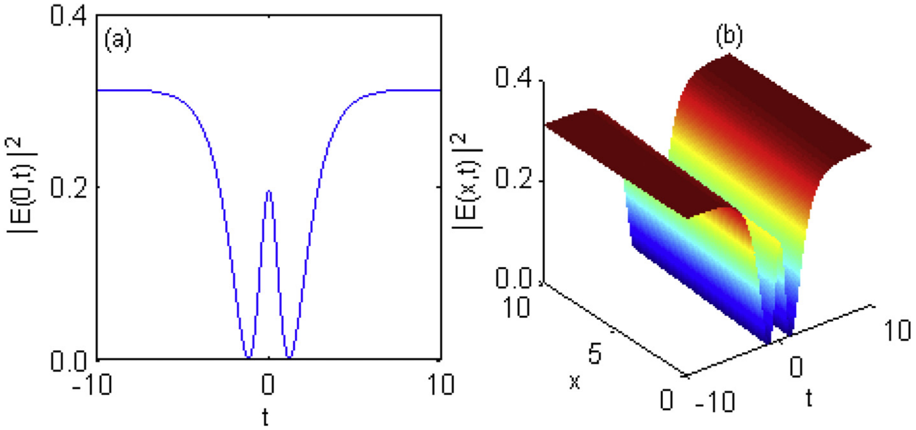

Fig. 2. (a) Intensity of the solitary wave profile $|E(0,t)|^2|$ as a function of t and its (b) evolution as computed from Eq. (25) for the values $\rho = \eta = 1$ and$\chi = 0.1$.

$$A(x,t) = \lambda \tanh[\eta(t-\chi x)] + i\rho sech[\eta(t-\chi x)] \tag{26}$$

which means that $\beta = 0$ in the ansatz solution (4). Then, from Eqs. (7a)-(7k), we can obtain the following expressions:

$$a_1 = \chi + \frac{2a_5}{3}\left(2\lambda^2+\rho^2\right), a_2 = \frac{\gamma\eta^2\left(11\lambda^2+\rho^2\right)}{\lambda^2-\rho^2}, a_3 = -\frac{18\gamma\eta^4\left(3\lambda^2+\rho^2\right)}{\left(\lambda^2-\rho^2\right)^2}, a_6 = \lambda^2 a_3, a_8 = -\frac{2a_5\left(\lambda^2-\rho^2\right)}{3\eta^2}, a_{10} = -\frac{48\gamma\eta^2}{\lambda^2-\rho^2}, a_{12} = \frac{12\gamma\eta^2}{\lambda^2-\rho^2}, \tag{27}$$

which yield the solitary wave parameters:

$$\eta^2 = -\frac{r_4}{6}\left(\rho^2-\lambda^2\right) \tag{28}$$

$$\chi = -\Omega + 3\alpha_3\Omega^2 + 4\gamma\Omega^3 + \frac{r_4}{6}(\alpha_3+4\gamma\Omega)\left(2\lambda^2+\rho^2\right) \tag{29}$$

$$k = -\frac{1}{2}\Omega^2 + \alpha_3\Omega^3 + \gamma\Omega^4 - \frac{\gamma r_4^2}{2}\lambda^2\left(\rho^2+3\lambda^2\right) \tag{30}$$

$$\rho^2 = -\frac{22 + 9r_4 + 6\alpha_3\Omega\,[22(r_7-r_6) - 9r_4] + 2\gamma r_4\Omega^2}{8\gamma r_4^2}, \tag{31}$$

$$\lambda^2 = \frac{2 + 3r_4 + 6\alpha_3\Omega\,[2(r_7-r_6) - 3r_4] - 26\gamma r_4\Omega^2}{8\gamma r_4^2} \tag{32}$$

With

$$\Omega = \frac{(24r_7+r_4)\alpha_3}{44\gamma r_4} \tag{33}$$

Therefore, we found the solution for Eq. (1) as

$$E(x,t) = \{\lambda \tanh[\eta(t-\chi x)] + i\rho sech[\eta(t-\chi x)]\}\exp[i(kx-\Omega\, t)] \tag{34}$$

whose intensity is

$$|E(x,t)|^2 = \lambda^2 + \left(\rho^2-\lambda^2\right)sech^2[\eta(t-\chi x)] \tag{35}$$

For this case, we must have $r_1 = r_3 = r_5 = 0$ for the solution (34) to exist. Now from Eq. (33) and the condition $a_4 - 3a_5 = 0$, we can find another restriction on the model parameters as $33r_4(6r_7 - r_6) = (3r_4 - 4r_2)(24r_7 + r_4)$.

From Eq. (28), it is clear to see that if $r_4 < 0 (> 0)$, one must require $\rho^2 - \lambda^2 > 0$ (< 0), and the solution (34) represents a brightlike (darklike) solitary wave. Here we consider the special case of $r_4 > 0$ and $\rho^2 - \lambda^2 < 0$. In Fig. 3(a), we have presented

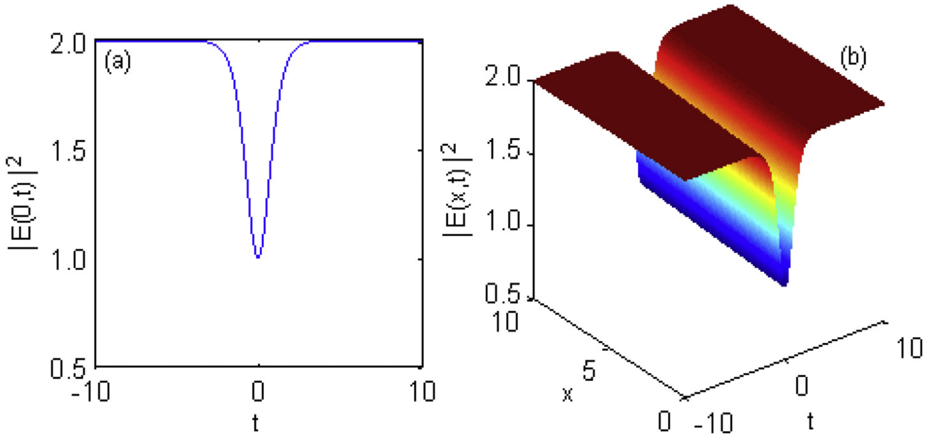

Fig. 3. (a) Intensity of the solitary wave profile $|E(0,t)|^2|$ as a function of t and its (b) evolution as computed from Eq. (35) for the values $\rho = \eta = 1$, $\lambda = \sqrt{2}$ and$\chi = 0.1$.

the intensity profile of the above solitary wave using the values $\rho = \eta = 1$, $\lambda = \sqrt{2}$, and$\chi = 0.1$, while Fig. 3(b) depicts its evolution. We can see that this solution describes the propagation of a gray solitary pulse (dark wave with nonzero minimum intensity) on a continuous-wave background, which does not change during the propagation distance as shown in Fig. 3(b).

4. Conclusion

In conclusion, we have considered an extended NLS equation with third- and fourth-order dispersion terms for describing ultrashort pulse propagation in highly dispersive media. Exact solitary wave solutions of the model have been derived using an ansatz method. The resulting solutions are in the form of bright, dark and W-shaped solitary waves. We have also presented the necessary conditions on material parameters for the formation of these localized structures. Considering the utility of the equation in fiber optics and other branches of physics, these solutions may find practical applications.

Future research may include a systematic study of the stability of the solitary waves presented here under the influence of finite perturbations by employing the numerical simulation methods. It would be also especially relevant to consider the same envelope equation but including distributed parameters to investigate femtosecond optical pulse propagation in inhomogeneous fiber media. Such studies are currently in progress and will be reported in future publications.

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