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Optical soliton perturbation in parabolic law medium having weak non-local nonlinearity by a couple of strategic integration architectures

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ABSTRACT

In this paper, the governing model with the inclusion of parabolic law nonlinearity, weakly non-local nonlinearity in addition to perturbation terms is examined for the sake of uncovering quite important optical soliton solutions. Dark, bright and singular solitons in addition to singular periodic solutions are yielded with the modified simple equation technique and trial equation architecture along with parameter restrictions.

1. Introduction

We know from technological developments that the optical soliton pulses form of the essential gradient for soliton communication technology. These pulses along optical waveguides are used in optical fibers, data transmission, telecommunications industry and transcontinental distances over the globe in a few seconds. The dynamical analysis of these pulses has increased the technology to the top level. These developments have prompted more comprehensive researches from the point of view of physical and engineering aspect. One of the important investigations in this fields is to study the considered governing model with optical nonlinearities The most important one of the governing models is the nonlinear Schrödinger equation (NLSE) which models light waves in optical fibers. Over the past few decades, NLSE has been intensely investigated in polarization preserving fibers, photonic crystal fibers, birefringent fibers and dense wavelength division multiplexing (DWDM) system along with Kerr law and non-Kerr law fiber nonlinearities [1–28]. In the open literature not only Kerr law fiber nonlinearity but also a variety of non-Kerr law fiber nonlinearities that are studied in various cases [1–27] such as power-law, quadratic–cubic law, parabolic-law, dual-power law, log-law, anti-cubic law, cubic-quintic-septic law, and triple-power law fiber nonlinearities [1]. We observe very recently -besides those nonlinearities- a new kind of non-Kerr law fiber nonlinearity which is called weak non-local law fiber nonlinearity [2–17]. From point of view of optics, nonlocality of nonlinearity means that the light-induced refractive index change of a material at a particular position is defined by the light intensity in a certain neighborhood of this position [12]. Nonlocality has an important impact on the propagation of beams and their localization. For instance, nonlocality can raise modulational instability in self-defocusing media, or suppress it in self-focusing media. Nonlocality may also suppress transverse instability of optical waves and prevent the catastrophic collapse of self-focusing beams in nonlinear media. Also, the nonlocal nonlinearity impacts the interactions between bright solitons as observed in experiments with lead glasses and nematic liquid crystal [12]. The nonlinear Schrödinger equations in Refs. [2–17] comprise only the usual group-velocity dispersion (GVD) and the parabolic law nonlinearity coupled with weakly non-local nonlinearity. GVD is an important feature of a dispersive medium and is used often to evaluate how the considered medium will impress an optical pulse traveling through it. The parabolic law fiber nonlinearity occurs in the nonlinear interaction between Langmuir waves and electrons. This nonlinearity describes the nonlinear interaction between the high frequency Langmuir waves and the ion-acoustic waves [18]. Unlike all of

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these studies in Refs. [2–17], the model will be studied from a different perspective in this work. This work will consider the governing model with the inclusion of the spatio-temporal dispersion term (STD), the GVD, the inter-modal dispersion, the self-steepening, the higher-order dispersions, the full nonlinearity and the parabolic law nonlinearity coupled with weakly non-local nonlinearity. STD is a union of the spatial dispersion and the temporal dispersion. The temporal dispersion usually occurred in optics describes the memory effects in the considered model. On the other hand, spatial dispersion defines spreading effects and is usually important only at small length scales and also assists quite small perturbations to optics. Thus, the inclusion of the STD term in the considered model prompts it into a generalized and well-posed problem for better understanding the optics perspective. In addition, in the model, the perturbation terms throughout the full nonlinearity parameters are considered for giving a generalized viewpoint. Because of these reasons, finding optical soliton pulses to the model in polarization preserving optical fibers becomes very important. To achieve this goal, the model is going to be investigated to obtain optical soliton pulses with the help of modified simple equation methodology [16,17] and trial equation approach [16,17,19,20]. These two schemes will reveal dark soliton, bright soliton with singular soliton type solutions that will be presented throughout their presence criterion. The details of the analysis are investigated in the progressive segments of this study.

1.1. Governing model

Throughout parabolic law, weakly non-local nonlinearity spatio-temporal dispersion term and perturbation terms, the governing model called the perturbed nonlocal nonlinear Schrödinger's equation (NNLSE) will be employed as follows

$$iq_t + aq_{xx} + bq_{xt} + (c_1 |q|^2 + c_2 |q|^4)q + c_3(|q|^2)_{xx}q = i[\rho q_x + \sigma(|q|^{2m}q)_x + \gamma(|q|^{2m})_x q]. \tag{1}$$

The first term on the left side of this model accounts for the temporal evolution of pulses and also the presence of spatio-temporal dispersion term and group velocity dispersion sequentially is provided by the coefficient of b and a. The complex valued function $q(x, t)$ with the inclusion of x, t independent variables stands for the wave profile. The coefficients of c_1 and c_2 are called as the nonlinear terms which respectively are refered to as cubic and quintic nonlinearities. Additionally, the coefficient of c_3 represents weakly non-local nonlinearity. The perturbation terms σ implies to self-steepening effect, ρ corresponds to the inter-modal dispersion and γ signifies the nonlinear dispersion. Finally, the full nonlinearity is given as the exponent m.

2. A quick skim through trial equation approach

A quick glance over the trial equation methodology [20,21] is yielded with a view to study quite important solitons.

Step-1: A nonlinear evolution equation can be given by

$$\Psi(\theta, \theta_t, \theta_x, \theta_{tt}, \theta_{xt}, \theta_{xx}, ...) = 0 \tag{2}$$

with the dependent function and its partial derivatives shown as $\theta, \theta_t, \theta_x, \theta_{tt}, \theta_{xt}, \theta_{xx}, ...$ and also this equation is decreased in

$$F(\Psi, \Psi', \Psi'', \Psi''', \cdots) = 0 \tag{3}$$

by using of the conversion

$$\theta(x, t) = \Psi(\varphi) \tag{4}$$

with

$$\varphi = x - vt. \tag{5}$$

The dependent function and its derivatives are given by $\Psi, \Psi', \Psi'', \Psi''', \cdots$ sequentially in the ordinary differential Eq. (3). The parameter v corresponds to the velocity of the soliton while the independent variables $x$and t represent spatial and temporal variables respectively.

Step-2: The ancillary equation which is the key point of the scheme is yield as

$$(\Psi')^2 = H(\Psi) = \sum_{i=0}^{N} \delta_i \Psi^i \tag{6}$$

having the essential constant coefficients $\delta_0, \delta_1, ..., \delta_N$.

Step-3: The application of Eq. (6) is attached to the N number which can be obtained with the aid of balancing rule in the ordinary differential Eq. (3).

Step-4: The overdeterminet equations are acquired with the aid of putting Eq. (6) in Eq. (3) and setting of the constant coefficients of the same functions namely Ψ^i, $i = 0, 1, 2, ...$to zero. Thus, the requisite constants $\delta_0, \delta_1, ..., \delta_N$ can be given with the aid of solving the equations.

Step-5: The following equation can be reached as

$$\pm(\varphi - \varphi_0) = \int \frac{d\Psi}{\sqrt{\sum_{i=0}^{N} \delta_i \Psi^i}} \tag{7}$$

by using of Eq. (6). The solitons to Eq. (2) are acquired if the discriminants of $H(\Psi)$in (7) is classified [19,20].

2.1. Implementation to the governing model

With a view to study quite important optical solitons with the governing model, the following transformation

$$q(x, t) = P(\xi)e^{i\phi(x,t)} \tag{8}$$

with

$$\xi = x - vt \tag{9}$$

will be considered. The function $P(\xi)$ means the amplitude component and the function $\varphi(x, t)$ signifies the phase component that can be supposed by

$$\phi(x, t) = -\kappa x + \omega t + \theta. \tag{10}$$

The parameter κ means soliton frequency and the parameter ω stands for soliton wave number whilst the parameter θ signifies soliton phase.

The imaginary part is recovered by

$$-2a\kappa + b\kappa v - \rho + b\omega - v - ((2m + 1)\sigma + 2m\gamma)P^{2m} = 0 \tag{11}$$

because of inserting Eq. (8) into Eq. (1). If we set the coefficients of the linearly independent functions to zero in the imaginary component, we can easily obtain the relationship between the self–steepening effect and the nonlinear dispersion along with the full nonlinearity

$$(2m + 1)\sigma + 2m\gamma = 0 \tag{12}$$

while the velocity is located

$$v = \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}. \tag{13}$$

We would like to emphasize that it is possible to apply the modified simple equation methodology and the trial equation approach without taking into account the restrictions mentioned in Eqs. (12) and (13) because Eq. (11) is not an ordinary differential equation. Moreover, the methods can only be applied to ordinary differential equations where the balancing rule is applied.

Also, if Eq. (8) is put in Eq. (1), the real part is acquired as

$$(a - bv)P'' + 2c_3P^2P'' - (\omega + a\kappa^2 - b\kappa\omega + \rho\kappa)P + c_1P^3 + c_2P^5 + 2c_3P(P')^2 - \kappa\sigma P^{2m+1} = 0. \tag{14}$$

In the rest of the article, we will consider Eq. (14) which is an ordinary differential equations where the balancing rule is applied.

Case-1:

Eq. (6) can be yield by

$$(P')^2 = \delta_0 + \delta_1 P + \delta_2 P^2 + \delta_3 P^3 + \delta_4 P^4 \tag{15}$$

because of $N = 4$ which can be obtained by using of balancing rule P^2P'' or $P(P')^2$ between P^5 throughout the full nonlinearity $m = 1$ in the ordinary differential Eq. (14).

The overdetermined equations are acquired as follows

P^5 Coeff.:

$$6c_3\delta_4 + c_2 = 0, \tag{16}$$

P^4 Coeff.:

$$5c_3\delta_3 = 0, \tag{17}$$

P^3 Coeff.:

$$2(a - bv)\delta_4 + 4c_3\delta_2 + c_1 - \kappa\sigma = 0, \tag{18}$$

P^2 Coeff.:

$$3(a - bv)\delta_3 + 6c_3\delta_1 = 0, \tag{19}$$

P Coeff.:

$$(a - bv)\delta_2 - (\omega + a\kappa^2 - b\kappa\omega + \rho\kappa) + 2c_3\delta_0 = 0, \tag{20}$$

P^0 Coeff.:

$$(a - bv)\delta_1 = 0 \tag{21}$$

because of putting Eq. (15) in Eq. (14) and setting of the constant coefficients of the same functions to zero. The following results are given by

$$\delta_0 = -\frac{1}{24c_3^3}(-12ac_3^2\kappa^2 + b^2c_2v^2 + 12bc_3^2\kappa\omega - 3bc_3\kappa\sigma v - 2abc_2v + 3ac_3\kappa\sigma + 3bc_1c_3v - 12c_3^2\kappa\rho + a^2c_2 - 3ac_1c_3 - 12c_3^2\omega),$$

$$\delta_1 = 0, \quad \delta_2 = \frac{-bc_2v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{12c_3^2},$$

$$\delta_3 = 0, \quad \delta_4 = -\frac{c_2}{6c_3}. \tag{22}$$

Through the medium of the following transformation

$$P = \pm\sqrt{(4\delta_4)^{-\frac{1}{3}}w}, \tag{23}$$

Eq. (15) can be transformed into the following integral

$$\pm(4\delta_4)^{\frac{1}{3}}(\xi - \xi_0) = \int \frac{dw}{\sqrt{w(w^2 + b_1w + b_0)}} \tag{24}$$

where

$$b_1 = 4\delta_2(4\delta_4)^{-\frac{2}{3}},$$

$$b_0 = 4\delta_0(4\delta_4)^{-\frac{1}{3}}. \tag{25}$$

According to Liu's method of complete discrimination for polynomial [19,20], we can solve the integral (24).

Type 1:

The solitons of the governing model are emerged as follows:

$$q(x, t) = \pm\sqrt{\frac{-bc_2v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{2c_2c_3}} \times \operatorname{sech}\left[\sqrt{\frac{-bc_2v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{12c_3^2}} \times\left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]e^{i(-\kappa x + \omega t + \theta)}. \tag{26}$$

The solution (26) points out bright soliton (see Figs. 1 and 2) provided that

$$\triangle = b_1^2 - 4b_0 > 0, \quad b_0 = 0,$$

$$w > -b_1, \quad \delta_2 > 0, \quad \delta_4 < 0. \tag{27}$$

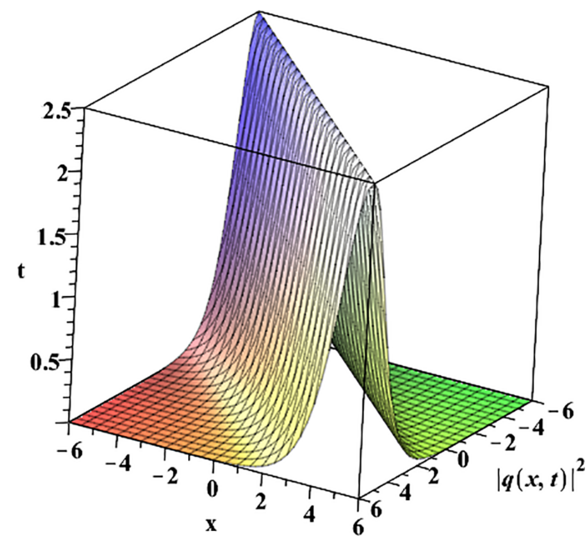

Fig. 1. 3D plot of the bright soliton (26) setting all arbitrary parameters to unity except $b = 2$, $c_1 = -1$.

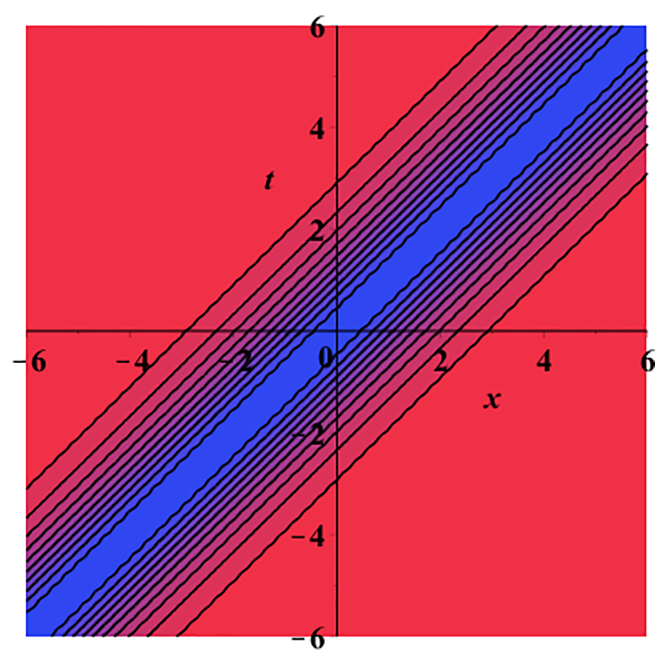

Fig. 2. Contour plot of $|q(x, t)|^2$ corresponding to the bright soliton (26) setting all arbitrary parameters to unity except $b = 2$, $c_1 = -1$.

$$q(x, t) = \pm\sqrt{-\frac{-bc_2v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{2c_2c_3}} \times \operatorname{csch}\left[\sqrt{\frac{-bc_2v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{12c_3^2}} \times\left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]e^{i(-\kappa x + \omega t + \theta)}. \tag{28}$$

The solution (28) points out singular soliton provided that

$$\triangle = b_1^2 - 4b_0 > 0, \quad b_0 = 0,$$

$$w > -b_1, \quad \delta_2 > 0, \quad \delta_4 > 0. \tag{29}$$

$$q(x, t) = \pm\sqrt{\frac{-bc_2v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{2c_2c_3}} \times \sec\left[\sqrt{-\frac{-bc_2v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{12c_3^2}} \times\left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]e^{i(-\kappa x + \omega t + \theta)}, \tag{30}$$

$$q(x,t) = \pm\sqrt{\frac{-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{2c_2c_3}} \times \csc\left[\sqrt{-\frac{-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{12c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)}. \tag{31}$$

The consequences (30) and (31) signify singular periodic solutions on condition that

$$\triangle = b_1^2 - 4b_0 > 0,\ \ b_0 = 0,$$

$$w > -b_1,\ \ \delta_2 < 0,\ \ \delta_4 > 0. \tag{32}$$

Type 2:

The solitons of the governing model are emerged as follows:

$$q(x,t) = \pm\sqrt{\frac{-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{4c_2c_3}} \times \tanh\left[\sqrt{-\frac{-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{24c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)}. \tag{33}$$

The solution (33) points out dark soliton (see Figs. 3 and 4) provided that

$$\triangle = b_1^2 - 4b_0 = 0,\quad w > 0,\quad \delta_2 < 0,\quad \delta_4 > 0,$$

$$\delta_0 = -\frac{(-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3)^2}{96c_2c_3^3}. \tag{34}$$

$$q(x,t) = \pm\sqrt{\frac{-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{4c_2c_3}} \times \coth\left[\sqrt{-\frac{-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{24c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)}. \tag{35}$$

The solution (35) points out singular soliton provided that

$$\triangle = b_1^2 - 4b_0 = 0,\quad w > 0,\quad \delta_2 < 0,\quad \delta_4 > 0,$$

$$\delta_0 = -\frac{(-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3)^2}{96c_2c_3^3}. \tag{36}$$

$$q(x,t) = \pm\sqrt{-\frac{-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{4c_2c_3}} \times \tan\left[\sqrt{\frac{-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{24c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{37}$$

$$q(x,t) = \pm\sqrt{-\frac{-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{4c_2c_3}} \times \cot\left[\sqrt{\frac{-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{24c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)}. \tag{38}$$

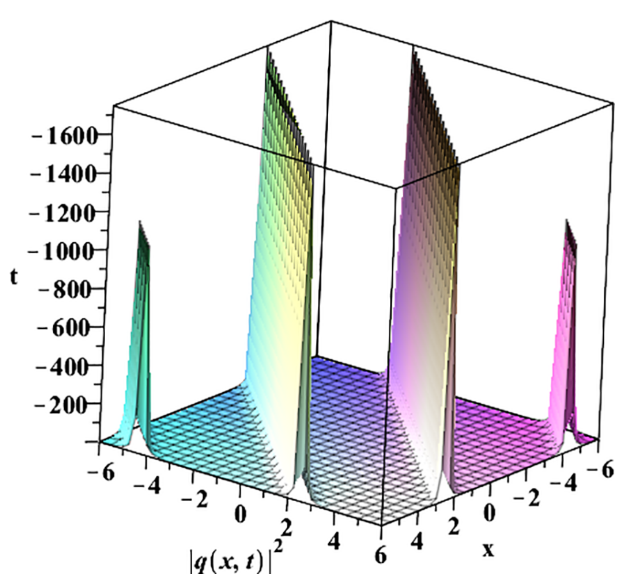

Fig. 3. 3D plot of the dark soliton (33) setting all arbitrary parameters to unity except $b = 2$.

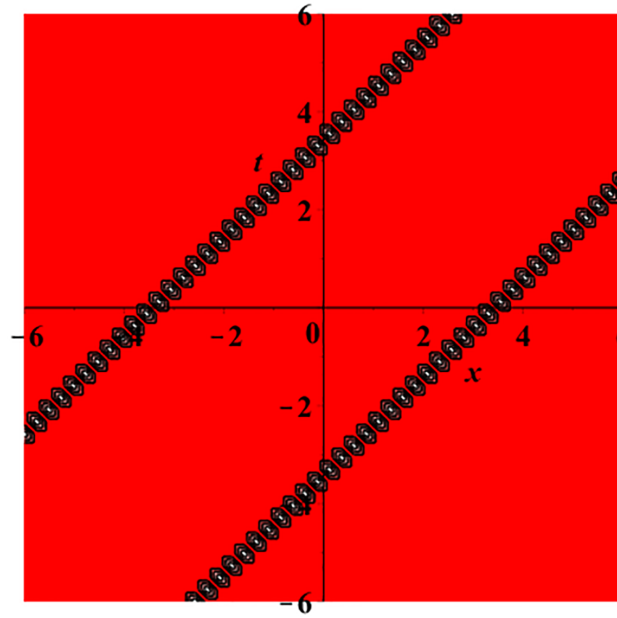

Fig. 4. Contour plot of $|q(x,t)|^2$ corresponding to the dark soliton (33) setting all arbitrary parameters to unity except $b = 2$.

The consequences (37) and (38) signify singular periodic solutions provided that

$$\triangle = b_1^2 - 4b_0 = 0,\quad w > 0,\quad \delta_2 > 0,\quad \delta_4 > 0,$$

$$\delta_0 = -\frac{(-bc_2\nu + 3c_3\kappa\sigma + ac_2 - 3c_1c_3)^2}{96c_2c_3^3}. \tag{39}$$

Case-2:

Eq. (14) can be yield by

$$(a - b\nu)(-(V')^2 + 2VV'') + 2c_3V(-(V')^2 + 2VV'') - 4(\omega + a\kappa^2 - b\kappa\omega + \rho\kappa) V^2 + 4c_1V^3 + 4c_2V^4 + 2c_3V(V')^2 - 4\kappa\sigma V^{m+2} = 0 \tag{40}$$

through the medium of the transformation $P = V^{\frac{1}{2}}$. Eq. (6) can be yield by

$$(V')^2 = \delta_0 + \delta_1V + \delta_2V^2 + \delta_3V^3 \tag{41}$$

because of $N = 3$ which can be obtained by using of balancing rule $V(V')^2$ or V^2V'' with V^4 throughout the full nonlinearity $m = 1$ in the ordinary differential Eq. (40),

The overdeterminet equations are acquired as follows

V^4 Coeff.:

$$6c_3\delta_3 + 4c_2 = 0, \tag{42}$$

V^3 Coeff.:

$$2(a - b\nu)\delta_3 + 4c_3\delta_2 + 4c_1 - 4\kappa\sigma = 0, \tag{43}$$

V^2 Coeff.:

$$(a - b\nu)\delta_2 - 4(\omega + a\kappa^2 - b\kappa\omega + \rho\kappa) + 2c_3\delta_1 = 0, \tag{44}$$

V^0 Coeff.:

$$(a - b\nu)\delta_0 = 0 \tag{45}$$

because of putting Eq. (41) in Eq. (40) and setting of the constant coefficients of the same functions to zero. The following results are given by

$$\delta_1 = -\frac{1}{6c_3^3}(-12ac_3^2\kappa^2 + b^2c_2\nu^2 + 12bc_3^2\kappa\omega - 3bc_3\kappa\sigma\nu - 2abc_2\nu + 3ac_3\kappa\sigma + 3bc_1c_3\nu - 12c_3^2\kappa\rho + a^2c_2 - 3ac_1c_3 - 12c_3^2\omega),$$

$$\delta_2 = \frac{-bc_2 v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{3c_3^2},$$

$$\delta_0 = 0, \quad \delta_3 = -\frac{2c_2}{3c_3}. \tag{46}$$

Through the medium of the following transformation

$$V = \pm\sqrt{(\delta_3)^{-\frac{1}{3}} w}, \tag{47}$$

Eq. (41) can be transformed into the following integral

$$\pm(\delta_3)^{\frac{1}{3}}(\xi - \xi_0) = \int \frac{dw}{\sqrt{w(w^2 + d_2 w + d_1)}} \tag{48}$$

where

$$d_2 = \delta_2(\delta_3)^{-\frac{2}{3}},$$

$$d_1 = \delta_1(\delta_3)^{-\frac{1}{3}}. \tag{49}$$

According to Liu's method of complete discrimination for polynomial [19,20], we can solve the integral (48).

The solitons of the governing model are emerged as follows:

$$q(x,t) = \left\{\frac{-bc_2 v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{2c_2c_3} \times \operatorname{sech}^2\left[\sqrt{\frac{-bc_2 v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{12c_3^2}} \times\left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}. \tag{50}$$

The solution (50) points out bright soliton provided that

$$\triangle = d_2^2 - 4d_1 > 0, \quad d_1 = 0,$$

$$w > -d_2, \quad \delta_2 > 0, \quad \delta_3 < 0. \tag{51}$$

$$q(x,t) = \left\{-\frac{-bc_2 v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{2c_2c_3} \times \operatorname{csch}^2\left[\sqrt{\frac{-bc_2 v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{12c_3^2}} \times\left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}. \tag{52}$$

The solution (52) points out singular soliton provided that

$$\triangle = d_2^2 - 4d_1 > 0, \quad d_1 = 0,$$

$$w > -d_2, \quad \delta_2 > 0, \quad \delta_3 > 0. \tag{53}$$

$$q(x,t) = \left\{-\frac{-bc_2 v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{2c_2c_3} \times \sec^2\left[\sqrt{-\frac{-bc_2 v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{12c_3^2}} \times\left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}, \tag{54}$$

$$q(x,t) = \left\{-\frac{-bc_2 v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{2c_2c_3} \times \csc^2\left[\sqrt{-\frac{-bc_2 v + 3c_3\kappa\sigma + ac_2 - 3c_1c_3}{12c_3^2}} \times\left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}. \tag{55}$$

The consequences (54) and (55) signify singular periodic solutions provided that

$$\triangle = d_2^2 - 4d_1 > 0, \quad d_1 = 0,$$

$$w > -d_2, \quad \delta_2 < 0, \quad \delta_3 < 0. \tag{56}$$

Case-3:

Eq. (6) can be yield by

$$(P')^2 = \delta_0 + \delta_1 P + \delta_2 P^2 + \delta_3 P^3 + \delta_4 P^4 \tag{57}$$

because of $N = 4$ which can be obtained by using of balancing rule P^2P'' or $P(P')^2$ between P^5 throughout the full nonlinearity $m = 2$ in the ordinary differential Eq. (14).

The overdeterminet equations are acquired as follows

P^5 Coeff.:

$$6c_3\delta_4 + c_2 - \kappa\sigma = 0, \tag{58}$$

P^4 Coeff.:

$$5c_3\delta_3 = 0, \tag{59}$$

P^3 Coeff.:

$$2(a - bv)\delta_4 + 4c_3\delta_2 + c_1 = 0, \tag{60}$$

P^2 Coeff.:

$$3(a - bv)\delta_3 + 6c_3\delta_1 = 0, \tag{61}$$

P Coeff.:

$$(a - bv)\delta_2 - (\omega + a\kappa^2 - b\kappa\omega + \rho\kappa) + 2c_3\delta_0 = 0, \tag{62}$$

P^0 Coeff.:

$$(a - bv)\delta_1 = 0 \tag{63}$$

because of putting Eq. (57) in Eq. (14) and setting of the constant coefficients of the same functions to zero. The following results are given by

$$\delta_0 = -\frac{1}{24c_3^3}(-b^2\kappa\sigma v^2 + 2ab\kappa\sigma v - 12ac_3^2\kappa^2 + b^2c_2v^2 + 12bc_3^2\kappa\omega - a^2\kappa\sigma - 2abc_2v + 3bc_1c_3 v - 12c_3^2\kappa\rho + a^2c_2 - 3ac_1c_3 - 12c_3^2\omega),$$

$$\delta_1 = 0, \quad \delta_2 = \frac{b\kappa\sigma v - a\kappa\sigma - bc_2v + ac_2 - 3c_1c_3}{12c_3^2},$$

$$\delta_3 = 0, \quad \delta_4 = \frac{\kappa\sigma - c_2}{6c_3}. \tag{64}$$

Through the medium of the following transformation

$$P = \pm\sqrt{(4\delta_4)^{-\frac{1}{3}} w}, \tag{65}$$

Eq. (57) can be transformed into the following integral

$$\pm(4\delta_4)^{\frac{1}{3}}(\xi - \xi_0) = \int \frac{dw}{\sqrt{w(w^2 + b_1 w + b_0)}} \tag{66}$$

where

$$b_1 = 4\delta_2(4\delta_4)^{-\frac{2}{3}},$$

$$b_0 = 4\delta_0(4\delta_4)^{-\frac{1}{3}}. \tag{67}$$

According to Liu's method of complete discrimination for polynomial [19,20], we can solve the integral (66).

Type 1:

The solitons of the governing model are emerged as follows:

$$q(x,t) = \pm\sqrt{-\frac{b\kappa\sigma v - a\kappa\sigma - bc_2v + ac_2 - 3c_1c_3}{2c_3(\kappa\sigma - c_2)}} \times \operatorname{sech}\left[\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bc_2v + ac_2 - 3c_1c_3}{12c_3^2}} \times\left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right] e^{i(-\kappa x + \omega t + \theta)}. \tag{68}$$

The solution (68) points out bright soliton provided that

$$\triangle = b_1^2 - 4b_0 > 0, \quad b_0 = 0,$$

$$w > -b_1, \quad \delta_2 > 0, \quad \delta_4 < 0. \tag{69}$$

$$q(x,t) = \pm\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bc_2v + ac_2 - 3c_1c_3}{2c_3(\kappa\sigma - c_2)}} \times \operatorname{csch}\left[\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bc_2v + ac_2 - 3c_1c_3}{12c_3^2}} \times\left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t\right)\right] e^{i(-\kappa x + \omega t + \theta)}. \tag{70}$$

The solution (70) points out singular soliton provided that

$$\triangle = b_1^2 - 4b_0 > 0, \quad b_0 = 0,$$

$$w > -b_1, \quad \delta_2 > 0, \quad \delta_4 > 0. \tag{71}$$

$$q(x,t)=\pm\sqrt{-\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{2c_3(\kappa\sigma-c_2)}}\times\sec\left[\sqrt{-\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{12c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)},\tag{72}$$

$$q(x,t)=\pm\sqrt{-\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{2c_3(\kappa\sigma-c_2)}}\times\csc\left[\sqrt{-\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{12c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)}.\tag{73}$$

where Eqs. (72) and (73) mean singular periodic solutions whenever

$$\triangle=b_1^2-4b_0>0,\quad b_0=0,$$

$$w>-b_1,\quad \delta_2<0,\quad \delta_4>0.\tag{74}$$

Type 2:

The solitons of the governing model are emerged as follows:

$$q(x,t)=\pm\sqrt{-\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{4c_3(\kappa\sigma-c_2)}}\times\tanh\left[\sqrt{-\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{24c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)}.\tag{75}$$

The solution (75) points out dark soliton provided that

$$\triangle=b_1^2-4b_0=0,\quad w>0,\quad \delta_2<0,\quad \delta_4>0$$

$$\delta_0=\frac{(b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3)^2}{96(\kappa\sigma-c_2)c_3^3}.\tag{76}$$

$$q(x,t)=\pm\sqrt{-\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{4c_3(\kappa\sigma-c_2)}}\times\coth\left[\sqrt{-\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{24c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)}.\tag{77}$$

The solution (77) points out singular soliton provided that

$$\triangle=b_1^2-4b_0=0,\quad w>0,\quad \delta_2<0,\quad \delta_4>0$$

$$\delta_0=\frac{(b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3)^2}{96(\kappa\sigma-c_2)c_3^3}.\tag{78}$$

$$q(x,t)=\pm\sqrt{\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{4c_3(\kappa\sigma-c_2)}}\times\tan\left[\sqrt{\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{24c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)},\tag{79}$$

$$q(x,t)=\pm\sqrt{\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{4c_3(\kappa\sigma-c_2)}}\times\cot\left[\sqrt{\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{24c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t\right)\right]e^{i(-\kappa x+\omega t+\theta)}.\tag{80}$$

The consequences (79) and (80) signify singular periodic solutions provided that

$$\triangle=b_1^2-4b_0=0,\quad w>0,\quad \delta_2>0,\quad \delta_4>0$$

$$\delta_0=\frac{(b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3)^2}{96(\kappa\sigma-c_2)c_3^3}.\tag{81}$$

Case-4:

Eq. (6) can be yield by

$$(V')^2=\delta_0+\delta_1V+\delta_2V^2+\delta_3V^3\tag{82}$$

because of $N=3$ which can be obtained by using of balancing rule $V(V')^2$ or V^2V'' with V^4 throughout the full nonlinearity $m=2$ in Eq. (40).

The overdetermined equations are acquired as follows

V^4 Coeff.:

$$6c_3\delta_3+4c_2-4\kappa\sigma=0,\tag{83}$$

V^3 Coeff.:

$$2(a-b\nu)\delta_3+4c_3\delta_2+4c_1=0,\tag{84}$$

V^2 Coeff.:

$$(a-b\nu)\delta_2-4(\omega+a\kappa^2-b\kappa\omega+\rho\kappa)+2c_3\delta_1=0,\tag{85}$$

V^0 Coeff.:

$$(a-b\nu)\delta_0=0\tag{86}$$

because of putting Eq. (82) in Eq. (40) and setting of the constant coefficients of the same functions to zero. The following results are given by

$$\delta_1=-\frac{1}{6c_3^3}(-b^2\kappa\sigma\nu^2+2ab\kappa\sigma\nu-12ac_3^2\kappa^2+b^2c_2\nu^2+12bc_3^2\kappa\omega-a^2\kappa\sigma-2abc_2\nu+3bc_1c_3\nu-12c_3^2\kappa\rho+a^2c_2-3ac_1c_3-12c_3^2\omega),$$

$$\delta_2=\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{3c_3^2},\quad \delta_0=0,\quad \delta_3=\frac{2(\kappa\sigma-c_2)}{3c_3}.\tag{87}$$

Through the medium of the following transformation

$$V=\pm\sqrt{(\delta_3)^{-\frac{1}{3}}w},\tag{88}$$

Eq. (82) can be transformed into the following integral

$$\pm(\delta_3)^{\frac{1}{3}}(\xi-\xi_0)=\int\frac{dw}{\sqrt{w(w^2+d_2w+d_1)}}\tag{89}$$

where

$$d_2=\delta_2(\delta_3)^{-\frac{2}{3}},$$

$$d_1=\delta_1(\delta_3)^{-\frac{1}{3}}.\tag{90}$$

According to Liu's method of complete discrimination for polynomial [19,20], we can solve the integral (89).

The solitons of the governing model are emerged as follows:

$$q(x,t)=\left\{-\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{2c_3(\kappa\sigma-c_2)}\times\text{sech}^2\left[\sqrt{\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{12c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t\right)\right]\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)}.\tag{91}$$

The solution (91) points out bright soliton provided that

$$\triangle=d_2^2-4d_1>0,\quad d_1=0,$$

$$w>-d_2,\quad \delta_2>0,\quad \delta_3<0.\tag{92}$$

$$q(x,t)=\left\{\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{2c_3(\kappa\sigma-c_2)}\times\text{csch}^2\left[\sqrt{\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{12c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t\right)\right]\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)}.\tag{93}$$

The solution (93) points out singular soliton provided that

$$\triangle=d_2^2-4d_1>0,\quad d_1=0,$$

$$w>-d_2,\quad \delta_2>0,\quad \delta_3>0.\tag{94}$$

$$q(x,t)=\left\{\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{2c_3(\kappa\sigma-c_2)}\times\sec^2\left[\sqrt{-\frac{b\kappa\sigma\nu-a\kappa\sigma-bc_2\nu+ac_2-3c_1c_3}{12c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t\right)\right]\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)},\tag{95}$$

$$q(x,\,t)=\left\{\frac{b\kappa\sigma v-a\kappa\sigma-bc_2v+ac_2-3c_1c_3}{2c_3(\kappa\sigma-c_2)}\times\csc^2\left[\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bc_2v+ac_2-3c_1c_3}{12c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t\right)\right]\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)}. \tag{96}$$

The consequences (95) and (96) signify singular periodic solutions provided that

$$\triangle=d_2^2-4d_1>0,\quad d_1=0,$$

$$w>-d_2,\quad \delta_2<0,\quad \delta_3<0. \tag{97}$$

3. A quick skim through modified simple equation approach

A quick glance over the modified simple equation methodology [18] is yield with a view to study quite important solitons.

Step-1: A nonlinear evolution equation can be given by

$$\Psi(\theta,\,\theta_t,\,\theta_x,\,\theta_{tt},\,\theta_{xt},\,\theta_{xx},\,...)=0 \tag{98}$$

with the dependent function and its partial derivatives shown as $\theta,\,\theta_t,\,\theta_x,\,\theta_{tt},\,\theta_{xt},\,\theta_{xx},\,...$and also this equation is decreased in

$$F(\Psi,\,\Psi',\,\Psi'',\,\Psi''',\,\cdots)=0 \tag{99}$$

by using of the conversion

$$\theta(x,\,t)=\Psi(\varphi) \tag{100}$$

with

$$\varphi=x-vt. \tag{101}$$

The dependent function and its derivatives are given by $\Psi,\,\Psi',\,\Psi'',\,\Psi''',\,\cdots$sequentially in the ordinary differential Eq. (99). The parameter v corresponds to the velocity of the soliton while the independent variables $x$and t represent spatial and temporal variables respectively.

Step-2: The ancillary equation which is the key point of the scheme is yield as

$$\Psi(\varphi)=\sum_{i=0}^{N}\delta_i\left(\frac{P'(\vartheta)}{P(\vartheta)}\right)^i \tag{102}$$

having the essential constant coefficients $\delta_0,\,\delta_1,...,\,\delta_N$.

Step-3: The application of Eq. (102) is attached to the N number which can be obtained with the aid of balancing rule in the ordinary differential Eq. (99).

Step-4: The overdeterminet equations are acquired with the aid of putting Eq. (102) in Eq. (99) and setting of the constant coefficients of the same functions namely $P^{-i}(\vartheta)$, $i=1,\,2,\,...$to zero. Thus, the explicit solutions to Eq. (98) are acquired if the requisite constants $\delta_0,\,\delta_1,...,\,\delta_N$ can be given with the aid of solving the equations.

3.1. Implementation to the governing model

The application of the approach is yield in this section with a view to study quite important solitons of the governing model.

Case-1:

Eq. (102) can be yield by

$$P(\xi)=\delta_0+\delta_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right) \tag{103}$$

because of $N=1$ which can be obtained by using of balancing rule P'' with P^3 throughout $m=1$ in the ordinary differential Eq. (14).

The overdeterminet equations are acquired as follows

ψ^{-5} coeff.:

$$\delta_1^3(\psi')^5(c_2\delta_1^2+6c_3)=0, \tag{104}$$

ψ^{-4} coeff.:

$$5\delta_1^2(\psi')^3((c_2\delta_0\delta_1^2+2c_3\delta_0)\psi'-2c_3\delta_1\psi'')=0, \tag{105}$$

ψ^{-3} coeff.:

$$\delta_1\psi'((10c_2\delta_0^2\delta_1^2-\kappa\sigma\delta_1^2+c_1\delta_1^2+4c_3\delta_0^2-2bv+2a)(\psi')^2-16c_3\delta_0\delta_1\psi'\psi''+2c_3\delta_1^2\psi'\psi'''+2c_3\delta_1^2(\psi'')^2)=0, \tag{106}$$

ψ^{-2} coeff.:

$$\delta_1((10c_2\delta_0^3\delta_1-3\kappa\sigma\delta_0\delta_1+3c_1\delta_0\delta_1)(\psi')^2+(-6c_3\delta_0^2+3bv-3a)\psi'\psi''+4c_3\delta_0\delta_1\psi'\psi'''+2c_3\delta_0\delta_1(\psi'')^2)=0, \tag{107}$$

ψ^{-1} coeff.:

$$\delta_1((5c_2\delta_0^4-3\kappa\sigma\delta_0^2-a\kappa^2+b\kappa\omega+3c_1\delta_0^2-\kappa\rho-\omega)\psi'+(2c_3\delta_0^2-bv+a)\psi''')=0, \tag{108}$$

ψ^0 coeff.:

$$\delta_0(c_2\delta_0^4-\kappa\sigma\delta_0^2-a\kappa^2+b\kappa\omega+c_1\delta_0^2-\kappa\rho-\omega)=0. \tag{109}$$

because of putting Eq. (103) in Eq. (14) and setting of the constant coefficients of the same functions to zero. The following results can be given by

$$\delta_1=\pm\sqrt{-\frac{6c_3}{c_2}},$$

$$\delta_0=\pm\sqrt{-\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{4c_2c_3}},$$

$$\omega=\frac{1}{16c_2c_3^2(1-b\kappa)}(-16a\kappa^2c_2c_3^2+b^2v^2c_2^2-2b\kappa\sigma vc_2c_3-3\kappa^2\sigma^2c_3^2-2abvc_2^2+2a\kappa\sigma c_2c_3+2bvc_1c_2c_3-16\kappa\rho c_2c_3^2+6\kappa\sigma c_1c_3^2+a^2c_2^2-2ac_1c_2c_3-3c_1^2c_3^2) \tag{110}$$

and

$$\psi''=\pm\sqrt{\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{6c_3^2}}\psi', \tag{111}$$

$$\psi'''=\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{6c_3^2}\psi'. \tag{112}$$

The following equations can be reached as

$$\psi'=\pm\sqrt{\frac{6c_3^2}{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}}\times k_1e^{\pm\sqrt{\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{6c_3^2}}\xi}, \tag{113}$$

and

$$\psi=\frac{6c_3^2}{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}\times k_1e^{\pm\sqrt{\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{6c_3^2}}\xi}+k_2 \tag{114}$$

with k_1 and k_2 integration constants by means of using Eqs. (111) and (112). The solitons of the governing model are emerged as follows

$$q(x,\,t)=\left\{\pm\sqrt{-\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{4c_2c_3}}\pm\sqrt{-\frac{6c_3}{c_2}}\times\left(\frac{\pm\sqrt{\frac{6c_3^2}{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}}\times k_1e^{\pm\sqrt{\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{6c_3^2}}(x-vt)}}{\frac{6c_3^2}{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}\times k_1e^{\pm\sqrt{\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{6c_3^2}}(x-vt)}+k_2}\right)\right\}\times e^{i(-\kappa x+\omega t+\theta)} \tag{115}$$

on account of inserting Eqs. (113) and (114) into Eq. (103). If we set

$$k_1=\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{6c_3^2}\times e^{\pm\sqrt{\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{6c_3^2}}\xi_0},\quad k_2=\pm1, \tag{116}$$

we get:

$$q(x,t) = \pm\sqrt{-\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{4c_2c_3}} \times \tanh\left[\sqrt{\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{24c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t + \xi_0\right)\right] e^{i(-\kappa x + \omega t + \theta)}. \tag{117}$$

The solution (117) points out dark soliton (see Figs. 5 and 6) provided that

$$bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3 > 0. \tag{118}$$

$$q(x,t) = \pm\sqrt{-\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{4c_2c_3}} \times \coth\left[\sqrt{\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{24c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t + \xi_0\right)\right] e^{i(-\kappa x + \omega t + \theta)}. \tag{119}$$

The solution (119) points out singular soliton provided that

$$bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3 > 0. \tag{120}$$

$$q(x,t) = \pm\sqrt{\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{4c_2c_3}} \times \tan\left[\sqrt{-\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{24c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t + \xi_0\right)\right] e^{i(-\kappa x + \omega t + \theta)}, \tag{121}$$

$$q(x,t) = \pm\sqrt{\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{4c_2c_3}} \times \cot\left[\sqrt{-\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{24c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t + \xi_0\right)\right] e^{i(-\kappa x + \omega t + \theta)} \tag{122}$$

where Eqs. (121) and (122) mean singular periodic solutions whenever

$$bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3 < 0. \tag{123}$$

Case-2:

Eq. (102) can be yield by

$$V(\xi) = \delta_0 + \delta_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right) + \delta_2\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^2 \tag{124}$$

because of $N = 2$ which can be obtained by using of balancing rule $(V')^2$ or VV'' with V^3 throughout the full nonlinearity $m = 1$ in the ordinary differential Eq. (40).

The overdetermined equations are acquired as follows

ψ^{-8} coeff.:

$$4\delta_2^3(\psi')^8(c_2\delta_2 + 6c_3) = 0, \tag{125}$$

ψ^{-7} coeff.:

$$8\delta_2^2(\psi')^6((2c_2\delta_1\delta_2 + 7c_3\delta_1)\psi' - 5c_3\delta_2\psi'') = 0, \tag{126}$$

ψ^{-6} coeff.:

$$4\delta_2(\psi')^4((-\kappa\sigma\delta_2^2 + 4c_2\delta_0\delta_2^2 + 6c_2\delta_1^2\delta_2 - 2bv\delta_2 + c_1\delta_2^2 + 12c_3\delta_0\delta_2 + 10c_3\delta_1^2 + 2a\delta_2)(\psi')^2 - 23c_3\delta_1\delta_2\psi'\psi'' + 2c_3\delta_2^2\psi'\psi''' + 2c_3\delta_2^2(\psi'')^2) = 0, \tag{127}$$

ψ^{-5} coeff.:

$$4(\psi')^3((-3\kappa\sigma\delta_1\delta_2^2 + 12c_2\delta_0\delta_1\delta_2^2 + 4c_2\delta_1^3\delta_2 - 3bv\delta_1\delta_2 + 3c_1\delta_1\delta_2^2 + 16c_3\delta_0\delta_1\delta_2 + 2c_3\delta_1^3 + 3a\delta_1\delta_2)(\psi')^2 + (3bv\delta_2^2 - 20c_3\delta_0\delta_2^2 - 16c_3\delta_1^2\delta_2 - 3a\delta_2^2)\psi'\psi'' + 5c_3\delta_1\delta_2^2\psi'\psi''' + 4c_3\delta_1\delta_2^2(\psi'')^2) = 0, \tag{128}$$

ψ^{-4} coeff.:

$$-(\psi')^2((12\delta_0\delta_2 bv - 4\delta_2^2 b\kappa\omega + 12\delta_0\delta_2^2\kappa\sigma + 12\delta_1^2\delta_2\kappa\sigma - 48c_2\delta_0\delta_1^2\delta_2 - 12\delta_0\delta_2 a + 3\delta_1^2 bv - 24c_3\delta_0^2\delta_2 - 16c_3\delta_0\delta_1^2 + 4\delta_2^2 a\kappa^2 + 4\delta_2^2\rho\kappa - 12\delta_0\delta_2^2 c_1 - 12\delta_1^2\delta_2 c_1 - 24c_2\delta_0^2\delta_2^2 - 4c_2\delta_1^4 - 3\delta_1^2 a + 4\delta_2^2\omega)(\psi')^2 + (-18bv\delta_1\delta_2 + 104c_3\delta_0\delta_1\delta_2 + 12c_3\delta_1^3 + 18a\delta_1\delta_2)\psi'\psi'' + (4bv\delta_2^2 - 16c_3\delta_0\delta_2^2 - 16c_3\delta_1^2\delta_2 - 4a\delta_2^2)\psi'\psi''' + (-16c_3\delta_0\delta_2^2 - 8c_3\delta_1^2\delta_2)(\psi'')^2) = 0, \tag{129}$$

ψ^{-3} coeff.:

$$-2\psi'((4a\kappa^2\delta_1\delta_2 - 4b\kappa\omega\delta_1\delta_2 + 12\kappa\sigma\delta_0\delta_1\delta_2 + 2\kappa\sigma\delta_1^3 - 24c_2\delta_0^2\delta_1\delta_2 - 8c_2\delta_0\delta_1^3 + 2bv\delta_0\delta_1 + 4\kappa\rho\delta_1\delta_2 - 12c_1\delta_0\delta_1\delta_2 - 2c_1\delta_1^3 - 4c_3\delta_0^2\delta_1 - 2a\delta_0\delta_1 + 4\omega\delta_1\delta_2)(\psi')^2 + (-10bv\delta_0\delta_2 - 2bv\delta_1^2 + 20c_3\delta_0^2\delta_2 + 12c_3\delta_0\delta_1^2 + 10a\delta_0\delta_2 + 2a\delta_1^2)\psi'\psi'' + (3bv\delta_1\delta_2 - 12c_3\delta_0\delta_1\delta_2 - 2c_3\delta_1^3 - 3a\delta_1\delta_2)\psi'\psi''' - 8c_3\delta_0\delta_1\delta_2(\psi'')^2) = 0, \tag{130}$$

ψ^{-2} coeff.:

$$(-8a\kappa^2\delta_0\delta_2 - 4a\kappa^2\delta_1^2 + 8b\kappa\omega\delta_0\delta_2 + 4b\kappa\omega\delta_1^2 - 12\kappa\sigma\delta_0^2\delta_2 - 12\kappa\sigma\delta_0\delta_1^2 + 16c_2\delta_0^3\delta_2 + 24c_2\delta_0^2\delta_1^2 - 8\kappa\rho\delta_0\delta_2 - 4\kappa\rho\delta_1^2 + 12c_1\delta_0^2\delta_2 + 12c_1\delta_0\delta_1^2 - 8\omega\delta_0\delta_2 - 4\omega\delta_1^2)(\psi')^2 + (6bv\delta_0\delta_1 - 12c_3\delta_0^2\delta_1 - 6a\delta_0\delta_1)\psi'\psi'' + (-4bv\delta_0\delta_2 - 2bv\delta_1^2 + 8c_3\delta_0^2\delta_2 + 8c_3\delta_0\delta_1^2 + 4a\delta_0\delta_2 + 2a\delta_1^2)\psi'\psi''' + (-4bv\delta_0\delta_2 + bv\delta_1^2 + 8c_3\delta_0^2\delta_2 + 4a\delta_0\delta_2 - a\delta_1^2)(\psi'')^2 = 0, \tag{131}$$

ψ^{-1} coeff.:

$$-2\delta_0\delta_1((4a\kappa^2 - 4b\kappa\omega + 6\kappa\sigma\delta_0 - 8c_2\delta_0^2 + 4\kappa\rho - 6c_1\delta_0 + 4\omega)\psi' + (bv - 2c_3\delta_0 - a)\psi''') = 0, \tag{132}$$

ψ^0 coeff.:

$$-4\delta_0^2(a\kappa^2 - b\kappa\omega + \kappa\sigma\delta_0 - c_2\delta_0^2 + \kappa\rho - c_1\delta_0 + \omega) = 0 \tag{133}$$

because of putting Eq. (124) in Eq. (40) and setting of the constant coefficients of the same functions to zero. The following results are given by

$$\delta_1 = \pm\sqrt{-\frac{12(bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3)}{c_2^2}},\ \delta_0 = 0,$$

$$\delta_2 = -\frac{6c_3}{c_2},\ \omega = \frac{1}{12c_3^2(1 - b\kappa)}(-12a\kappa^2c_3^2 + b^2v^2c_2 - 3b\kappa\sigma vc_3 - 2abvc_2 + 3a\kappa\sigma c_3 + 3bvc_1c_3 - 12\kappa\rho c_3^2 + a^2c_2 - 3ac_1c_3) \tag{134}$$

and

$$\psi'' = \pm\sqrt{-\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{3c_3^2}}\,\psi', \tag{135}$$

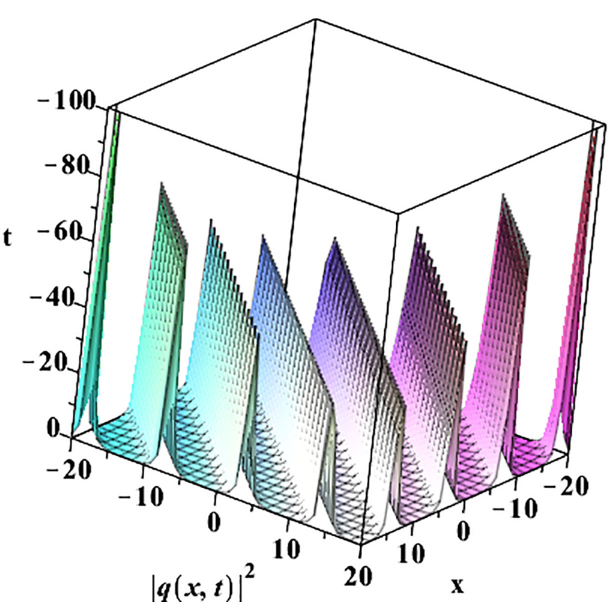

Fig. 5. 3D plot of the dark soliton (117) setting all arbitrary parameters to unity except$b = 2$.

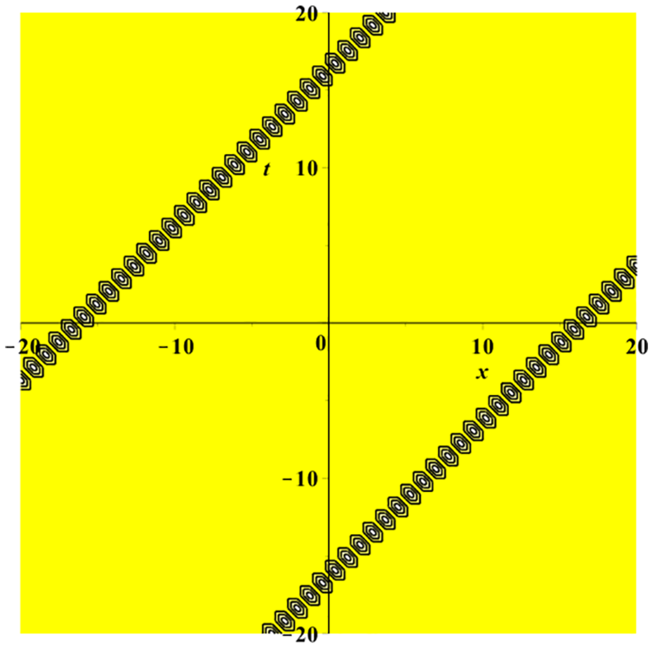

Fig. 6. Contour plot of $|q(x, t)|^2$ corresponding to the dark soliton (117) setting all arbitrary parameters to unity except $b = 2$.

$$\psi''' = -\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{3c_3^2}\psi'. \tag{136}$$

The following equations can be reached as

$$\psi' = \pm\sqrt{-\frac{3c_3^2}{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}} \times k_1 e^{\pm\sqrt{-\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{3c_3^2}}\xi}, \tag{137}$$

and

$$\psi = -\frac{3c_3^2}{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3} \times k_1 e^{\pm\sqrt{-\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{3c_3^2}}\xi} + k_2 \tag{138}$$

with k_1 and k_2 integration constants by means of using Eqs. (135) and (136). The solitons of the governing model are emerged as follows

$$q(x, t) = \left\{\pm\sqrt{-\frac{12(bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3)}{c_2^2}} \times \left(\frac{\pm\sqrt{-\frac{3c_3^2}{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}} \times k_1 e^{\pm\sqrt{-\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{3c_3^2}}(x-vt)}}{-\frac{3c_3^2}{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3} \times k_1 e^{\pm\sqrt{-\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{3c_3^2}}(x-vt)} + k_2}\right) - \frac{6c_3}{c_2} \times \left(\frac{\pm\sqrt{-\frac{3c_3^2}{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}} \times k_1 e^{\pm\sqrt{-\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{3c_3^2}}(x-vt)}}{-\frac{3c_3^2}{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3} \times k_1 e^{\pm\sqrt{-\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{3c_3^2}}(x-vt)} + k_2}\right)^2\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\theta)} \tag{139}$$

on account of inserting Eqs. (137) and (138) into Eq. (124). If we set

$$k_1 = -\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{3c_3^2} \times e^{\pm\sqrt{-\frac{bvc_2-3\kappa\sigma c_3-ac_2+3c_1c_3}{3c_3^2}}\xi_0}, \quad k_2 = \pm 1, \tag{140}$$

we get:

$$q(x, t) = \left\{-\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{2c_2c_3} \times \operatorname{sech}^2\left[\sqrt{-\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{12c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t + \xi_0\right)\right]\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\theta)}. \tag{141}$$

The solution (141) points out bright soliton (see Figs. 7 and 8) provided that

$$bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3 < 0. \tag{142}$$

$$q(x, t) = \left\{\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{2c_2c_3} \times \operatorname{csch}^2\left[\sqrt{-\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{12c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t + \xi_0\right)\right]\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\theta)}. \tag{143}$$

The solution (143) points out singular soliton provided that

$$bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3 < 0. \tag{144}$$

$$q(x, t) = \left\{-\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{2c_2c_3} \times \sec^2\left[\sqrt{\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{12c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t + \xi_0\right)\right]\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\theta)}, \tag{145}$$

$$q(x, t) = \left\{-\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{2c_2c_3} \times \csc^2\left[\sqrt{\frac{bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3}{12c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t + \xi_0\right)\right]\right\}^{\frac{1}{2}} \times e^{i(-\kappa x+\omega t+\theta)} \tag{146}$$

where Eqs. (145) and (146) mean singular periodic solutions whenever

$$bvc_2 - 3\kappa\sigma c_3 - ac_2 + 3c_1c_3 > 0. \tag{147}$$

Case-3:

Eq. (102) can be yield by

$$P(\xi) = \delta_0 + \delta_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right) \tag{148}$$

because of $N = 1$ which can be obtained by using of balancing rule P'' with P^3 throughout the full nonlinearity $m = 2$ in the ordinary differential Eq. (14).

The overdetermined equations are acquired as follows

ψ^{-5} coeff.:

$$-\delta_1^3(\psi')^5(\kappa\sigma\delta_1^2 - c_2\delta_1^2 - 6c_3) = 0, \tag{149}$$

ψ^{-4} coeff.:

$$-5\delta_1^2(\psi')^3((\kappa\sigma\delta_0\delta_1^2 - c_2\delta_0\delta_1^2 - 2c_3\delta_0)\psi' + 2c_3\delta_1\psi'') = 0, \tag{150}$$

ψ^{-3} coeff.:

$$\delta_1\psi'((-10\kappa\sigma\delta_0^2\delta_1^2+10c_2\delta_0^2\delta_1^2+c_1\delta_1^2+4c_3\delta_0^2-2bv+2a)(\psi')^2-16c_3\delta_0\delta_1\psi'\psi''+2c_3\delta_1^2\psi'\psi'''+2c_3\delta_1^2(\psi'')^2)=0, \tag{151}$$

ψ^{-2} coeff.:

$$-\delta_1((10\kappa\sigma\delta_0^3\delta_1-10c_2\delta_0^3\delta_1-3c_1\delta_0\delta_1)(\psi')^2+(6c_3\delta_0^2-3bv+3a)\psi'\psi''-4c_3\delta_0\delta_1\psi'\psi'''-2c_3\delta_0\delta_1(\psi'')^2)=0, \tag{152}$$

ψ^{-1} coeff.:

$$-\delta_1((5\kappa\sigma\delta_0^4-5c_2\delta_0^4+a\kappa^2-b\kappa\omega-3c_1\delta_0^2+\kappa\rho+\omega)\psi'+(-2c_3\delta_0^2+bv-a)\psi''')=0, \tag{153}$$

ψ^0 coeff.:

$$\delta_0(-\kappa\sigma\delta_0^4+c_2\delta_0^4-a\kappa^2+b\kappa\omega+c_1\delta_0^2-\kappa\rho-\omega)=0 \tag{154}$$

because of putting Eq. (148) in Eq. (14) and setting of the constant coefficients of the same functions to zero. The following results can be given by

$$\delta_0=\pm\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{4c_3(\kappa\sigma-c_2)}},$$

$$\delta_1=\pm\sqrt{\frac{6c_3}{\kappa\sigma-c_2}},$$

$$\omega=\frac{1}{16(\kappa\sigma-c_2)c_3^2(b\kappa-1)}(b^2\kappa^2\sigma^2v^2-2ab\kappa^2\sigma^2v+16a\kappa^3\sigma c_3^2-2b^2\kappa\sigma v^2c_2+a^2\kappa^2\sigma^2+4ab\kappa\sigma vc_2-16a\kappa^2c_2c_3^2+b^2v^2c_2^2-2b\kappa\sigma vc_1c_3+16\kappa^2\rho\sigma c_3^2-2a^2\kappa\sigma c_2-2abvc_2^2+2a\kappa\sigma c_1c_3+2bvc_1c_2c_3-16\kappa\rho c_2c_3^2+a^2c_2^2-2ac_1c_2c_3-3c_1^2c_3^2)$$

and

$$\psi''=\pm\sqrt{\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{-6c_3^2}}\psi', \tag{155}$$

$$\psi'''=\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{-6c_3^2}\psi'. \tag{156}$$

The following equations can be reached as

$$\psi'=\pm\sqrt{-\frac{6c_3^2}{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}}\times k_1e^{\pm\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{6c_3^2}}\xi}, \tag{157}$$

and

$$\psi=-\frac{6c_3^2}{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}\times k_1e^{\pm\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{6c_3^2}}\xi}+k_2 \tag{158}$$

with k_1 and k_2 integration constants by means of using Eqs. (155) and (156). The solitons of the governing model are emerged as follows

$$q(x,t)=\left\{\pm\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{4c_3(\kappa\sigma-c_2)}}\pm\sqrt{\frac{6c_3}{\kappa\sigma-c_2}}\times\left(\frac{\pm\sqrt{-\frac{6c_3^2}{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}}\times k_1e^{\pm\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{6c_3^2}}(x-vt)}}{-\frac{6c_3^2}{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}\times k_1e^{\pm\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{6c_3^2}}(x-vt)}+k_2}\right)\right\}\times e^{i(-\kappa x+\omega t+\theta)} \tag{159}$$

on account of inserting Eqs. (157) and (158) into Eq. (148). If we set

$$k_1=-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{6c_3^2}\times e^{\pm\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{6c_3^2}}\xi_0},$$
$$k_2=\pm1, \tag{160}$$

we get:

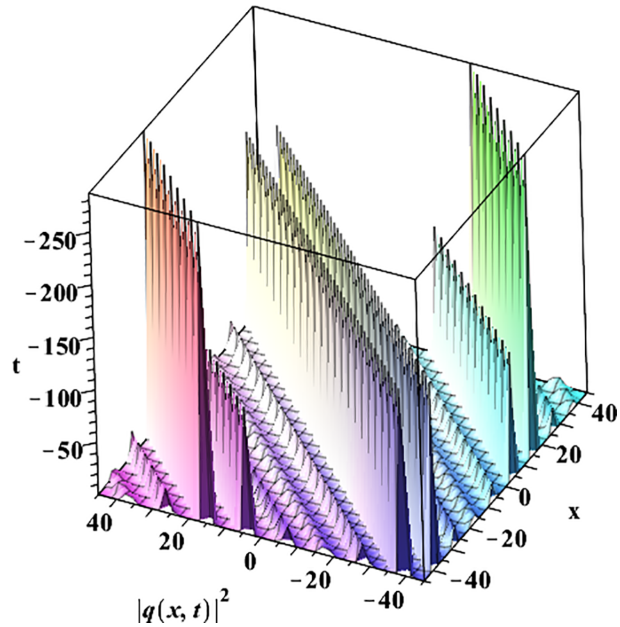

Fig. 7. 3D plot of the bright soliton (141) setting all arbitrary parameters to unity except $b=2$, $c_1=-1$.

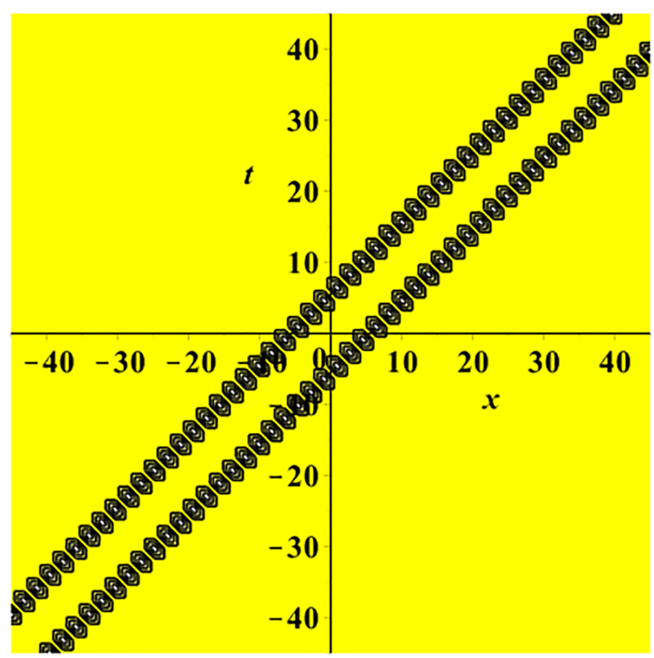

Fig. 8. Contour plot of $|q(x,t)|^2$ corresponding to the bright soliton (141) setting all arbitrary parameters to unity except $b=2$, $c_1=-1$.

$$q(x,t)=\pm\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{4c_3(\kappa\sigma-c_2)}}\times\tanh\left[\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{24c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t+\xi_0\right)\right]e^{i(-\kappa x+\omega t+\theta)}. \tag{161}$$

The solution (161) points out dark soliton provided that

$$b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3<0. \tag{162}$$

$$q(x,t)=\pm\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{4c_3(\kappa\sigma-c_2)}}\times\coth\left[\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{24c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t+\xi_0\right)\right]e^{i(-\kappa x+\omega t+\theta)}. \tag{163}$$

The solution (163) points out singular soliton provided that

$$b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3 < 0. \tag{164}$$

$$q(x,t) = \pm\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}{4c_3(\kappa\sigma - c_2)}} \times \tan\left[\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}{24c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t + \xi_0\right)\right] e^{i(-\kappa x + \omega t + \theta)}, \tag{165}$$

$$q(x,t) = \pm\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}{4c_3(\kappa\sigma - c_2)}} \times \cot\left[\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}{24c_3^2}} \times \left(x - \frac{2a\kappa + \rho - b\omega}{b\kappa - 1}t + \xi_0\right)\right] e^{i(-\kappa x + \omega t + \theta)} \tag{166}$$

where Eqs. (165) and (166) mean singular periodic solutions whenever

$$b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3 > 0. \tag{167}$$

Case-4:

Eq. (102) can be yield by

$$V(\xi) = \delta_0 + \delta_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right) + \delta_2\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^2 \tag{168}$$

because of $N = 2$ which can be obtained by using of balancing rule $(V')^2$ or VV'' with V^3 throughout the full nonlinearity $m = 2$ in the ordinary differential Eq. (40).

The overdetermined equations are acquired as follows

ψ^{-8} coeff.:

$$-4\delta_2^3(\psi')^8(\kappa\sigma\delta_2 - c_2\delta_2 - 6c_3) = 0, \tag{169}$$

ψ^{-7} coeff.:

$$-8\delta_2^2(\psi')^6((2\kappa\sigma\delta_1\delta_2 - 2c_2\delta_1\delta_2 - 7c_3\delta_1)\psi' + 5c_3\delta_2\psi'') = 0, \tag{170}$$

ψ^{-6} coeff.:

$$4\delta_2(\psi')^4((-4\kappa\sigma\delta_0\delta_2^2 - 6\kappa\sigma\delta_1^2\delta_2 + 4c_2\delta_0\delta_2^2 + 6c_2\delta_1^2\delta_2 - 2bv\delta_2 + c_1\delta_2^2 + 12c_3\delta_0\delta_2 + 10c_3\delta_1^2 + 2a\delta_2)(\psi')^2 - 23c_3\delta_1\delta_2\psi'\psi'' + 2c_3\delta_2^2\psi'\psi''' + 2c_3\delta_2^2(\psi'')^2) = 0, \tag{171}$$

ψ^{-5} coeff.:

$$4(\psi')^3((-12\kappa\sigma\delta_0\delta_1\delta_2^2 - 4\kappa\sigma\delta_1^3\delta_2 + 12c_2\delta_0\delta_1\delta_2^2 + 4c_2\delta_1^3\delta_2 - 3bv\delta_1\delta_2 + 3c_1\delta_1\delta_2^2 + 16c_3\delta_0\delta_1\delta_2 + 2c_3\delta_1^3 + 3a\delta_1\delta_2)(\psi')^2 + (3bv\delta_2^2 - 20c_3\delta_0\delta_2^2 - 16c_3\delta_1^2\delta_2 - 3a\delta_2^2)\psi'\psi'' + 5c_3\delta_1\delta_2^2\psi'\psi''' + 4c_3\delta_1\delta_2^2(\psi'')^2) = 0, \tag{172}$$

ψ^{-4} coeff.:

$$-(\psi')^2((48\delta_0\delta_1^2\delta_2\kappa\sigma + 12\delta_0\delta_2 bv - 4\delta_2^2 b\kappa\omega - 48\delta_0\delta_1^2\delta_2 c_2 + 24\delta_0^2\delta_2^2\kappa\sigma - 12\delta_0\delta_2 a + 3\delta_1^2 bv - 24c_3\delta_0^2\delta_2 - 16c_3\delta_0\delta_1^2 + 4\delta_2^2 a\kappa^2 + 4\delta_2^2\rho\kappa - 12c_1\delta_0\delta_2^2 - 12c_1\delta_1^2\delta_2 - 24\delta_0^2\delta_2^2 c_2 + 4\delta_1^4\kappa\sigma + 4\delta_2^2\omega - 4\delta_1^4 c_2 - 3\delta_1^2 a)(\psi')^2 + (-18bv\delta_1\delta_2 + 104c_3\delta_0\delta_1\delta_2 + 12c_3\delta_1^3 + 18a\delta_1\delta_2)\psi'\psi'' + (4bv\delta_2^2 - 16c_3\delta_0\delta_2^2 - 16c_3\delta_1^2\delta_2 - 4a\delta_2^2)\psi'\psi''' + (-16c_3\delta_0\delta_2^2 - 8c_3\delta_1^2\delta_2)(\psi'')^2) = 0, \tag{173}$$

ψ^{-3} coeff.:

$$-2\psi'((24\kappa\sigma\delta_0^2\delta_1\delta_2 + 8\kappa\sigma\delta_0\delta_1^3 + 4a\kappa^2\delta_1\delta_2 - 4b\kappa\omega\delta_1\delta_2 - 24c_2\delta_0^2\delta_1\delta_2 - 8c_2\delta_0\delta_1^3 + 2bv\delta_0\delta_1 + 4\kappa\rho\delta_1\delta_2 - 12c_1\delta_0\delta_1\delta_2 - 2c_1\delta_1^3 - 4c_3\delta_0^2\delta_1 - 2a\delta_0\delta_1 + 4\omega\delta_1\delta_2)(\psi')^2 + (-10bv\delta_0\delta_2 - 2bv\delta_1^2 + 20c_3\delta_0^2\delta_2 + 12c_3\delta_0\delta_1^2 + 10a\delta_0\delta_2 + 2a\delta_1^2)\psi'\psi'' + (3bv\delta_1\delta_2 - 12c_3\delta_0\delta_1\delta_2 - 2c_3\delta_1^3 - 3a\delta_1\delta_2)\psi'\psi''' - 8c_3\delta_0\delta_1\delta_2(\psi'')^2) = 0, \tag{174}$$

ψ^{-2} coeff.:

$$(-16\kappa\sigma\delta_0^3\delta_2 - 24\kappa\sigma\delta_0^2\delta_1^2 - 8a\kappa^2\delta_0\delta_2 - 4a\kappa^2\delta_1^2 + 8b\kappa\omega\delta_0\delta_2 + 4b\kappa\omega\delta_1^2 + 16c_2\delta_0^3\delta_2 + 24c_2\delta_0^2\delta_1^2 - 8\kappa\rho\delta_0\delta_2 - 4\kappa\rho\delta_1^2 + 12c_1\delta_0^2\delta_2 + 12c_1\delta_0\delta_1^2 - 8\omega\delta_0\delta_2 - 4\omega\delta_1^2)(\psi')^2 + (6bv\delta_0\delta_1 - 12c_3\delta_0^2\delta_1 - 6a\delta_0\delta_1)\psi'\psi'' + (-4bv\delta_0\delta_2 - 2bv\delta_1^2 + 8c_3\delta_0^2\delta_2 + 8c_3\delta_0\delta_1^2 + 4a\delta_0\delta_2 + 2a\delta_1^2)\psi'\psi''' + (-4bv\delta_0\delta_2 + bv\delta_1^2 + 8c_3\delta_0^2\delta_2 + 4a\delta_0\delta_2 - a\delta_1^2)(\psi'')^2 = 0, \tag{175}$$

ψ^{-1} coeff.:

$$-2\delta_0\delta_1((8\kappa\sigma\delta_0^2 + 4a\kappa^2 - 4b\kappa\omega - 8c_2\delta_0^2 + 4\kappa\rho - 6c_1\delta_0 + 4\omega)\psi' + (bv - 2c_3\delta_0 - a)\psi''') = 0, \tag{176}$$

ψ^{0} coeff.:

$$4\delta_0^2(-\kappa\sigma\delta_0^2 - a\kappa^2 + b\kappa\omega + c_2\delta_0^2 - \kappa\rho + c_1\delta_0 - \omega) = 0 \tag{177}$$

because of putting Eq. (168) in Eq. (40) and setting of the constant coefficients of the same functions to zero. The following results are given by

$$\delta_1 = \pm\sqrt{\frac{12(b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3)}{(\kappa\sigma - c_2)^2}},$$

$$\delta_0 = 0, \quad \delta_2 = \frac{6c_3}{\kappa\sigma - c_2},$$

$$\omega = \frac{1}{12c_3^2(b\kappa - 1)}(b^2\kappa\sigma v^2 - 2ab\kappa\sigma v + 12a\kappa^2c_3^2 - b^2v^2c_2 + a^2\kappa\sigma + 2abvc_2 - 3bvc_1c_3 + 12\kappa\rho c_3^2 - a^2c_2 + 3ac_1c_3) \tag{178}$$

and

$$\psi'' = \pm\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}{3c_3^2}}\,\psi', \tag{179}$$

$$\psi''' = \frac{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}{3c_3^2}\psi'. \tag{180}$$

The following equations can be reached as

$$\psi' = \pm\sqrt{\frac{3c_3^2}{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}} \times k_1 e^{\pm\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}{3c_3^2}}\xi}, \tag{181}$$

and

$$\psi = \frac{3c_3^2}{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3} \times k_1 e^{\pm\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}{3c_3^2}}\xi} + k_2 \tag{182}$$

with k_1 and k_2 integration constants by means of using Eqs. (179) and (180). The solitons of the governing model are emerged as follows

$$q(x,t) = \left\{\pm\sqrt{\frac{12(b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3)}{(\kappa\sigma - c_2)^2}} \times \left(\frac{\pm\sqrt{\frac{3c_3^2}{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}} \times k_1 e^{\pm\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}{3c_3^2}}(x - vt)}}{\frac{3c_3^2}{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3} \times k_1 e^{\pm\sqrt{\frac{b\kappa\sigma v - a\kappa\sigma - bvc_2 + ac_2 - 3c_1c_3}{3c_3^2}}(x - vt)} + k_2}\right)\right.$$

$$+\frac{6c_3}{\kappa\sigma-c_2}\times\left(\frac{\pm\sqrt{\frac{3c_3^2}{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}}\times k_1 e^{\pm\sqrt{\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{3c_3^2}}(x-vt)}}{\frac{3c_3^2}{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}\times k_1 e^{\pm\sqrt{\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{3c_3^2}}(x-vt)}+k_2}\right)^{2}\Bigg\}^{\frac{1}{2}}\times e^{i(-\kappa x+\omega t+\theta)} \tag{183}$$

on account of inserting Eqs. (181) and (182) into Eq. (168). If we set

$$k_1=\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{3c_3^2}\times e^{\pm\sqrt{\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{3c_3^2}}\xi_0},$$
$$k_2=\pm 1, \tag{184}$$

we get:

$$q(x,t)=\left\{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{2c_3(\kappa\sigma-c_2)}\times\operatorname{sech}^2\left[\sqrt{\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{12c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t+\xi_0\right)\right]\right\}^{\frac{1}{2}}\times e^{i(-\kappa x+\omega t+\theta)}. \tag{185}$$

The solution (185) points out bright soliton provided that

$$b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3>0. \tag{186}$$

$$q(x,t)=\left\{\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{2c_3(\kappa\sigma-c_2)}\times\operatorname{csch}^2\left[\sqrt{\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{12c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t+\xi_0\right)\right]\right\}^{\frac{1}{2}}\times e^{i(-\kappa x+\omega t+\theta)}. \tag{187}$$

The solution (187) points out singular soliton provided that

$$b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3>0. \tag{188}$$

$$q(x,t)=\left\{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{2c_3(\kappa\sigma-c_2)}\times\sec^2\left[\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{12c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t+\xi_0\right)\right]\right\}^{\frac{1}{2}}\times e^{i(-\kappa x+\omega t+\theta)}, \tag{189}$$

$$q(x,t)=\left\{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{2c_3(\kappa\sigma-c_2)}\times\csc^2\left[\sqrt{-\frac{b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3}{12c_3^2}}\times\left(x-\frac{2a\kappa+\rho-b\omega}{b\kappa-1}t+\xi_0\right)\right]\right\}^{\frac{1}{2}}\times e^{i(-\kappa x+\omega t+\theta)} \tag{190}$$

The consequences (189) and (190) signify singular periodic solutions provided that

$$b\kappa\sigma v-a\kappa\sigma-bvc_2+ac_2-3c_1c_3<0. \tag{191}$$

4. Conclusions

The governing model with the inclusion of parabolic law nonlinearity, weakly non-local nonlinearity, spatio-temporal dispersion term in addition to perturbation terms was examined for the sake of uncovering quite important optical soliton solutions. Dark, bright and singular solitons in addition to singular periodic solutions were yield with the modified simple equation technique and trial equation architecture along with parameter restrictions. Some graphics have been added to better understand the physical characteristics of the obtained bright soliton and dark soliton that are quite well known as optical soliton molecules or pulses in the literature. Comparing our work with the results of [2–17], Eq. (1) has not been discussed before in literature and thus our article is very novel study. The models in Refs. [2–17] include only the usual group-velocity dispersion and the parabolic law nonlinearity coupled with weakly non-local nonlinearity while this article includes the spatio-temporal dispersion term, the usual group-velocity dispersion, the inter-modal dispersion, the self-steepening for short pulses, the higher-order dispersion, the full nonlinearity and the parabolic law nonlinearity coupled with weakly non-local nonlinearity. When compared the methods, the methods can only be applied to ordinary differential equations which have the principle of balancing. Moreover, the methods cause optical dark, bright and singular soliton solutions in addition to singular periodic solutions along with parameter restrictions. For the full nonlinearity $m = 1$ and $m = 2$, optical bright, dark and singular soliton solutions are emerged from Eq. (14) by using of the trial equation approach while the only optical dark and singular soliton solutions are emerged from Eq. (14) by using of the modified simple equation technique. Also, the only optical bright and singular soliton solutions are emerged from Eq. (40) by the methods. Consequently, the trial equation architecture offers more optical soliton solution types than the modified simple equation technique by using of Eq. (14). The consequences acquired in this paper causes to consider the model as more elaborate. The solutions obtained by the two strategic methods show that the model is an integrable equation. For this reason, the model can be considered with weakly non-local nonlinearity tied to non-Kerr type nonlinearities such as power-law, quadratic-cubic law, dual-power law, log-law, anti-cubic law, cubic-quintic-septic law and triple-power law fiber nonlinearities. Also, the model can be extended to not only birefringent fibers with four-wave mixing (FWM) but also dense wavelength division multiplexed (DWDM) system. Thus, these new and strategic models in communication technology can be integrated by using of the two strategic methods and finally optical soliton solutions can be obtained by the two strategic methods. The details of this very valuable work mentioned in this section is going to be presented respectively.

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